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INTERDISCIPLINARY PERSPECTIVES OF APPLYING MATHEMATICS IN HIGH SCHOOL, REAL PROFILE

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Abstract. Analyzing the concerns of human society, we come to the imminent conclusion that mathematics has a particularly important role in all its spheres, leading to the formation of logical - abstract thinking, to the modeling of natural phenomena, to the systematization and ranking of priorities, as well as to the development of algorithms - basis for the other sciences. The continuous need to discover the analysis models of some phenomena led various researchers towards the improvement of classical mathematics (metrics), coming to be dominated by the algebraic side (the science of the discontinuous) combined with the topological one (the science of the continuum) as a response to the combined character of reality. G. Moisil mentioned "...next to (classical) metrical mathematics, logical structural mathematics was elaborated. Modern mathematics is a science dominated by the category of structure" [1]. According to Bellman, the proof of knowledge of phenomena is the precision of their anticipation, a fact based on formulas and mathematical calculations, and Mircea Malița defines the model as "a mental or written, qualitative or mathematical representation of a part of a reality that constitutes a system", which leads to the idea of its practical and experimental applicability. This article aims to exemplify some practical ways of using mathematics in various disciplines as well as to capitalize on the practical side in several other fields.

Keywords: interdisciplinarity, mathematical modeling, fields of application of mathematics, school curriculum, interdisciplinary applications.

PERSPECTIVE INTERDISCIPLINARE DE APLICARE A MATEMATICII LA LICEU, PROFIL REAL

Rezumat. Analizând preocupările societății umane, ajungem la concluzia iminentă că matematica are un rol deosebit de important în toate sferele acesteia, conducând la formarea unei gândiri logico – abstractă, la modelarea unor fenomene naturale, la sistematizarea și ierarhizarea priorităților, precum și la dezvoltarea unor algoritmi – bază pentru celelalte științe. Nevoia continuă de a descoperi modelele de analiză a unor fenomene au condus diverși cercetători spre perfecționarea matematicii clasice (metrică), ajungând să fie dominată de latura algebrică (știința discontinuului) combinată cu cea topologică (știința continuului) ca răspuns la caracterul combinat al realității. G. Moisil menționa *„...pe lânga matematica metrică (clasică) s-a elaborat matematica structurală logică. Matematica moderna este o știință dominată de categoria structurii"* [1]. În accepțiunea lui Bellman, dovada cunoașterii fenomenelor o constituie precizia anticipației acestora, fapt ce are ca bază formule și calcule matematice, iar Mircea Malița definește modelul ca *„o reprezentare mintală sau scrisă, calitativă sau matematică a unei părți dintr-o realitate ce constituie un sistem*”, ceea ce conduce la ideea de aplicabilitate practică și experimentală a acesteia. Articolul de față își propune să exemplifice câteva modalități practice de utilizare a matematicii în diverse discipline precum și să valorifice latura practică în alte câteva domenii.

Cuvinte-cheie: interdisciplinaritate, modelare matematică, domenii de aplicare ale matematicii, programă școlară, aplicații interdisciplinare.

I. Introduction

Interdisciplinarity can be approached as a reflection on its methodological, theoretical and institutional impact in the teaching-learning educational process. Due to the relevance of this concept, the way it is understood has expanded, generating both supporting works [13] and critical works, which emphasize the separation of disciplines [15]. Researchers have proposed various common, unifying themes between disciplines, such as linguistic structuralism or Marxism, concluding that "disciplinarity is a way of life, a mental approach that combines curiosity with openness and the desire to explore...". "It is done collectively... proving that there cannot be a separation between education and research" [4, p. 285].

Authors such as Asa Briggs of the University of Sussex and Guy Michaud of the University of Paris argued that, due to the multiple connections between methods, disciplines and various linguistic issues, an interdisciplinary approach is not only beneficial, but becomes a necessity. They argued that this approach should be implemented even in the institutional structure of a university, either by establishing dedicated departments or by creating an entire university with a specific interdisciplinary profile [4, p 253-257].

"Learning strategies, sometimes also called cognitive strategies, include repetition, elaboration, organization and metacognition to evaluate and regulate one's thinking" [18]. In Crowl's statements we find that everything that involves analyzing, systematizing and applying performative knowledge is higher order thinking, which can be amplified through various school or extracurricular activities.

"Interdisciplinarity has been defined differently in this century: as a methodology, concept, process, way of thinking, philosophy and reflexive ideology. It has been linked to attempts to expose the dangers of fragmentation, to restore old connections, to explore emerging relationships, and to create new subjects suitable to meet our practical and conceptual needs...A long-term reconceptualization of epistemology is necessary to capitalize and establish a common term" [21, p.196].

In this sense, we can see that mathematics is a universal language, which transcends the boundaries of traditional disciplines, by connecting abstract concepts and theories with the indispensable tool part for analysis, modeling and problem solving in all other fields. It is often considered "the language and tool of the sciences" due to its ability to accurately describe natural phenomena and provide a rigorous foundation for exploring various fields of knowledge. So, in the last decade it has far exceeded the boundaries of the natural sciences, "infiltrating" humanities, art, social sciences, etc.

Taking a brief look back, it can be seen that since 1997, the Academies in Europe and the United States have placed a special emphasis on interdisciplinarity.

1997 – The European Academy and the European Commission organize a conference in Cambridge on "Interdisciplinarity and the organization of knowledge in Europe".

1999 – Proceedings of the previous conference are published under the same title, with 24 contributors from 11 countries, 9 from the UK, some referencing Erich Jantsch's seminal 1972 OECD article [10].

1999 – John Ziman argues that "the drive toward ever greater specialization has to do with a scholarly demand for originality." It's easier to be a "big frog in a small pond" [39, pp. 74-75]. At the end of the essay, he supports the idea that "disciplines represent stability and uniformity", and "disciplinarity is a code word for diversity and adaptability" [39, pp. 81-82].

2000 - some researchers noticed the narrowing of unidisciplinary fields and published "Beyond Boundaries: Disciplines, Paradigms, and Theoretical Integration in International Studies" [32].

2001 - Miller in a review published in the Newsletter of the Association for Interdisciplinary Studies sees that this book does not respect the basic idea of discussing paradigms, the meanings of disciplines and their theoretical integration, but has the idea of overcoming disciplinary boundaries and cognitive mode to look at their usefulness.

2002 – Joe Moran publishes Interdisciplinarity, which focuses mainly on English cultural and linguistic studies, with interdisciplinary approaches [30].

2009 – Pami Aalto from the University of Tampere in Finland starts a project in which the issue of interdisciplinarity in relation to international studies is addressed, resulting in two books: "International Studies: Interdisciplinary Approaches" [1], and the second Global and regional issues: towards an interdisciplinary study [2]".

2010 – Oxford University Press publishes the Oxford Handbook of Interdisciplinarity [14], in which none of the 37 chapters refer to international studies, but only to some from the area.

2011 – Aalto and his colleagues make the claim: „We want to assert that International Studies – as a broader field of study than International Relations – must necessarily be more interdisciplinary than International Relations ever was during its heyday from the 1950s onwards " [1, p. 3], which they observed in the 1939 League of Nations publication University Teaching of International Relations [39], but also in Quincy Wright's work, "The study of International Relations (1955). Despite Wright's colossal efforts to synthesize and analyze more than 20 works from the fields of international studies, his influence on the subject was minimized by the Cold War and the emphasis at the time on interdisciplinarity, but a group of intellectuals from The University of Chicago, Hans Morgenthau, have tried to develop this subject through publications and international interdisciplinary studies [1, p. 11-19].

2013 – Andrew Barry and Georgina Born published a book in which they try to rethink and change the concept of interdisciplinarity, entitled: "Interdisciplinarity: Reconfigurations of Social and Natural Sciences", challenging the claim that "interdisciplinary activity is about combining and integrating knowledge from existing disciplines', proceeding on the premise that this accumulation and integration of knowledge comes from all existing sources, not just disciplines (such as community, local experience, indigenous knowledge, etc.). It states that the disciplines have the political advantage within the institutions and are the ones that control budgets and programs, determining the degree of involvement in research and teaching, but the truly interdisciplinary activities are those that are based on "responsibility, innovation, ontology" [7, p.41]. Also this year, various books were published, the first of which was: "Interdisciplinary Perspectives on International Law and International Relations: The State of the Art", edited by Jeffrey Dunoff and Mark Pollack (2013) - a more accurately translated title, according to the entire content, it would be "interdisciplinary perspectives on the historical relationship between international law and international relations" [12, p. 649].

2014 – The American Political Science Association established a task force to study interdisciplinary practice in 2007, and the report of this group was published under the title Interdisciplinarity: Its Role in a Discipline – Based Academy [3], within which the tension between the supporters of disciplinarity and those of interdisciplinarity can be found. Also here, four researchers and pioneers of interdisciplinarity: David Easton (systems), R. Duncan Luce (cognitive sciences), Susanne and Lloyd Rudolph (area studies), argue the merits of interdisciplinary sciences. Easton makes the statement that: „I don't see anything interesting that isn't interdisciplinary. I think the disciplines have exhausted their contributions to our understanding of politics" [3, p.55]. Lloyd concludes his interview: „I realize that not only do I value interdisciplinarity, but I also value being allowed to think outside the box of disciplinary methods. New concepts reveal new realities" [3, p. 72].

2015 – Patrick James and Steve Yetiv, publish: Advancing Interdisciplinary Approaches to International Relations, where they illustrate the application of multiple perspectives from different disciplines or situations to the subject of conflicts. Included here are political science, psychology, anthropology, gender studies, technology studies, history, demographics, and systems analysis [37, p. 324].

2016 – The British Academy, through a group chaired by David Soskice of the London School of Economics, publishes the report: "Crossing Paths: Interdisciplinary Institutions, Careers, Education and Applications", on interdisciplinary research and teaching in UK higher education United, and at the end of the report it states: "that the best way to promote interdisciplinarity is to support "strong disciplines" [35, p. 6]. The whole group also affirms the fact that: "the quality of interdisciplinary work consists in the way it brings the disciplines together" [35, p. 10], or shows that: "an understanding of the challenges of interdisciplinary integration, including methodological integration and the human side of promoting interactions and communication" leads to broad and truthful research [34, p. 70].

2017 – The manual appears in the second edition (Robert Froderman) where, within the 46 chapters, various problems are addressed, from finance to pedagogy, etc., the author claiming that: "interdisciplinarity is the bridge between academic sophists (disciplinary) and the rest of society" [14, p. 7].

II. Methodology

The way the current curriculum is structured, it offers students the opportunity to specialize in a single discipline, the notions from the other disciplines being only at the level of cultural amplification, which is why it is necessary to change it to an interdisciplinary level to facilitate the integration of information from various disciplines in application purpose or shaping a learning experience.

Analyzing the progressive way in which science evolves, it is found that the foundations of this program are laid by the structured and specific learning of each field, at the individual level, for the sedimentation of basic ideas, formulas and fundamental laws.

In a statement, Drivers and collaborators explained that interdisciplinary learning based on the nature of the sciences leads to numerous benefits:

- The process of understanding science is facilitated by its applicability.
- Students can make informed decisions about various socio-scientific issues.
- Children become aware of the need for in-depth learning of the contents.

Billingsley and Ramos reinforce the idea that the epistemic perspective of science teaching and learning based on its nature is the most important, based on "knowledge about knowing", but also that a recognition of the limitation of science is particularly important because it helps to explain and predicting physical, biological or chemical phenomena [19].

Lattuca highlights the interaction and integration that occurs between disciplines in the form of complementarity and shaping. He describes synthetic interdisciplinarity through research methods that combine theories and concepts, yet retain a clear, individual part of each discipline involved. This implies a unitary approach to the various processes and phenomena studied within several disciplines, with major implications on the curriculum design strategy, allowing the contextualization and application of the accumulated knowledge, thus becoming an active-formative component of the entire learning process. In this situation, the interdisciplinary activity becomes one that combines the cognitive side with the creative one, ensuring the selection, correlation of information, their understanding and processing with the aim of forming new capacities.

Although the foundations of interdisciplinarity are laid by scientific research, we can also discover ways to implement it at the level of the high school curriculum, both on the macro-educational side (that of didactic design - school curriculum, planning, lesson plans, school manual), and micro-educational (that of teaching-learning-evaluation didactic activities). The school content designed in this way must be based on the principle of an applied thinking that leads to concrete action, having an evolution arising from that of science, without leading to superficiality or the loss of veracity of monodisciplinary information [18, 19].

Some scientific accounts of the Zohar and Ben-David emphasize the "thinking behind the thinking" more than the "thinking behind the action", which leads to epistemic understanding and extensive visual representations of abstract notions.

Repko and Szostak evoke the idea of interdisciplinarity between similar disciplines, which are in constant change and tend to connect through all openings to research, and Frank and Gabler argue that interdisciplinarity reaches from the theoretical level to the level of knowledge and skills [18, 19].

Lattuca, Voight, and Fath state that "gathering and analyzing tasks associated with everyday problems that students face facilitate learning," and Hapern and Hagel suggest that the varied learning environment and heterogeneous conditions that lead them to "simulate unpredictable environments from "real" leads to an improvement in learning, especially when the knowledge acquired in an environment is applied in a diversified way. King and Kitchener support the idea that interdisciplinarity requires "reflective thinking" and well-structured, so that it leaves only room for positive interpretation, with various solutions, which correlates with creativity [18, 19].

From another perspective, De Jong and Ferguson – Hessler argue that interdisciplinarity is based on a structured algorithmization of memory, so that the information stored within it is systematized in units that relate for a good interpretation and solution of a problem [18, 19].

Interdisciplinary teaching is generally built around a specific topic or correlation of topics so that students focus better on understanding the correspondences between disciplines and their underlying logic, as opposed to unidisciplinary teaching that is focused on the subject matter.

Newell suggests that interdisciplinary teaching must be done gradually, to accustom students' thinking to this openness to disciplinary correlation and collaboration using the new cognitive base for the purpose of positive learning.

Spelled highlights the positive results at the middle level in a meta-analysis of interdisciplinary learning research, focusing on core knowledge within the domains involved, including disciplinary paradigms [18, 19].

In environments where learning is interdisciplinary, students rely on acquired disciplinary knowledge, so a balance between interdisciplinarity and unidisciplinarity links the curricular content facilitating a broad base of notions for a safe path in education.

Boix Mansilla and Duraisingh demonstrated with the help of university programs, "practical solutions where students strategically borrowed from different disciplines to create a viable and coherent way to approach a defined problem" [18, 19, p. 226].

Within the "Mathematics and Sciences" curricular area, interdisciplinarity is mandatory, taking into account the direct applicability of the disciplines involved (physics, chemistry, biology and mathematics), leading to an overall view of the dynamics of the studied phenomena with the possibility of generalization their. In general, mathematics within the other disciplines is a working tool (theorizing, formulating laws, models, etc.), theoretically outpacing the other sciences through the process of abstraction. The totality of the transformations that matter undergoes, both at the level of structure and at the level of energy, constitute levers for investigating the ways of using it, which is why abstract modeling is necessary, implicitly also the methods of teaching and achieving perfectible interdisciplinarity.

For example, a mathematical equation can describe a chemical or physical law, a biological or mechanical process, etc.. Physical phenomena are often explained with the help of chemistry, and calculations involve mathematics: the electrification of bodies, their conductivity and insulation, the notion of field, the production of electric current, is carried out with the involvement of the study of the matter from which they are made, of the chemical properties or of the galvanic elements.

Erduran and Dagher highlight the fact that all theoretical knowledge is valuable only when a connection of them is made based on common ideas, materializing through applicability over time. An example from which Erduran starts is the model of the atom, explaining that the atomic theory, the periodic law or the molecular theory have different meanings depending on the discipline studied: in physics it can only be reduced to a variation of energy, in chemistry to a structure of substance and in mathematics to be the result of calculations and formulas. However, Christie states that some of the physical laws can be axiomatizations of some mathematical formulas, while in chemistry some laws or theories can be approximations reducible to mathematical calculations, but there are also phenomena that cannot be exploited in this way, leading to true paradigms [18, 19].

The previous examples come to support the fact that an interdisciplinary teaching can be achieved when working in a team, something also affirmed by Dempsey "disappearing objectives related to content, process, identity and relationship contribute to conflict" [18, 19, p. 157], or when the teacher has several specializations, which are rare cases [18, 19].

In the situation where interdisciplinary teaching would be carried out in a team, experts such as Davis and Lattuca propose the development of criteria to establish the level of collaboration in interdisciplinary teaching (something that can also be adapted in the pre-university situation) [18, 19]:

- Each teacher should work on the part of content specific to his discipline, in order to achieve a connection between content and learning methods.
- Integrating contents so that the presentation from different disciplinary perspectives is fair and accessible to students.
- The teaching should be carried out so that the topics are approached from all aspects, capitalizing on their practical and theoretical usefulness.
- Testing and evaluation to provide measurable results both at the level of the formation of students' skills (through performance indicators and descriptors), as well as the degree of teacher involvement through specific or other feedback techniques (for example, according to the SMART model, etc.) to achieve the proposed objectives.
- Creating a specific frame of reference for teaching – learning – interdisciplinary assessment [18, 19].

Referring to the study of the educational impact of interdisciplinarity, Boix Mansilla and Duraising (2007) define interdisciplinary understanding as„ the ability to integrate knowledge and ways of thinking in two or more established disciplines for an advanced cognitive process. This process could be explaining a phenomenon, solving a problem, or creating a product, which would be impossible at the disciplinary level" [18, 19, p. 219] and which require the integration of knowledge from various fields.

They also present a frame of reference:

- „the disciplinary basis – the degree of work of the pupils/students based on a carefully selected discipline;
- integration - with the aim of favoring students' understanding;
- critical awareness – the curriculum should clearly present purpose and reflexivity [18, 19, din bibliografie 34, p.222].

In the case of mathematics, interdisciplinarity plays an essential role in exploring and understanding its application in various fields through the process of integration, such as science, technology, economics, medicine, biology, sports, religion, music and more, to gain a more comprehensive perspective on a problem or an object. Thus critical thinking skills, problem solving and practical application of knowledge can be developed.

• biology and genetics: mathematics is used to model and understand biological processes, such as the growth of populations and the evolution of species. Alan Turing and John von Neumann made significant contributions in this field.

• cryptography and computer security: mathematics is essential in developing encryption algorithms and protecting sensitive data. Pioneers like Alan Turing and Claude Shannon had a major impact in this field.

• physics and astronomy: mathematics is essential in understanding and describing physical and astronomical phenomena. For example, Albert Einstein's theory of relativity and Isaac Newton's equations were developed using complex mathematical concepts.

• economics and finance: mathematics is used in the analysis and modeling of financial markets, risk and investment. John Nash and Harry Markowitz made significant contributions in this field.

• engineering: mathematics is essential in the design and analysis of structures, in the modeling and simulation of systems and in the optimization of technical processes.

• computer science and artificial intelligence: mathematics is used in developing computational algorithms, analyzing data and understanding machine learning models.

• statistics and data analysis: mathematics is used to collect, analyze and interpret data in various fields.

• design and architecture: mathematics is used in the design of structures and forms, in the calculation of the strength of materials and in the optimization of spaces and architectural plans.

• medicine and bioinformatics: mathematics is used in the modeling and simulation of biological processes, in the analysis of medical data and in the development of algorithms for diagnosis and treatment.

• transport and road logistics: mathematics is used to optimize transport routes, to plan appointments and to efficiently manage the vehicle fleet.

• production and industrial engineering: mathematics is used in the optimization of production processes, in resource planning and in the analysis of the efficiency of industrial operations.

• financial mathematics: mathematics is used in assessing risk, modeling financial markets and developing investment strategies.

• games and simulations: mathematics is used in the development of algorithms for video games, in virtual simulations and in the analysis of game strategies.

• scientific research: mathematics is used in the analysis of experimental data, in the modeling of natural phenomena and in the development of scientific theories.

• applied mathematics in sociology and social sciences: mathematics is used in the analysis of social networks, in the modeling of human behavior and in the study of social interactions.

• astronomy and astrophysics: mathematics is used in modeling and simulating the motions of planets and celestial bodies, analyzing astronomical data, and developing cosmological theories.

• mathematics in music: mathematics is used in the analysis of musical structures, in chord theory, and in the development of algorithms for music composition and generation.

• mathematics in sports: mathematics is used in the analysis of sports statistics, in the modeling of sports performance and in the development of training and competition strategies.

• mathematics in meteorology: mathematics is used in the modeling and simulation of meteorological phenomena, in the analysis of meteorological data.

In the following I will present some examples of interdisciplinary applications in the field of genetics processed and remade after the books of the researchers: Hartl, D. L., & Clark, A. G; Freeman, S., & Herron, J.C.; Pierce, B.A. (2013); Gillespie, J.H.; Hardy, G.H.; Weinberg, W.; Hedrick, P.W.; and others for a thorough understanding of the mathematical notions involved in the study of population genetics.

III. Examples of interdisciplinary applications

Population genetics is a branch of biology that studies genetic variation in populations and how it changes over time. Key concepts include:

1. **Allele frequency**: represents the proportion of different variants of a gene in a population. For example, if the P allele has a frequency of 0.7 and the p allele has a frequency of 0.3, these are the respective frequencies of these alleles in a given population.
2. **Hardy-Weinberg Equilibrium**: A mathematical theory that describes how allele and genotype frequencies are expected to remain constant in a large, ideal population in the absence of other evolutionary forces such as mutation, migration, natural selection, and mating -random.
3. **Natural Selection**: the process by which traits that confer an advantage in survival and reproduction become more common in a population.
4. **Genetic Drift**: Random changes in gene frequencies that are more apparent in small populations, caused by statistical fluctuations rather than natural selection.
5. **Gene Flow (Migration)**: the transfer of genes between populations through the migration of individuals or gametes, influencing allele frequencies.
6. **Mutations**: Permanent changes in DNA that introduce new genetic variants and can affect long-term allele frequencies.
7. **Speciation**: The process by which populations separate genetically and evolve into distinct species, often due to the accumulation of genetic differences.

***Example* 1. Allele frequency**

It represents the proportion of a particular allele at a specific locus in a population. This is a fundamental concept in genetics and evolution because it allows researchers to understand the genetic diversity and mechanisms of natural selection in a population.

Mathematical definition

The allelic frequency p for an A allele is calculated as:

$$f_1(A) = \frac{the\ number\ of\ copies\ of\ the\ A\ allele\ in\ the\ population}{the\ total\ number\ of\ copies\ of\ all\ alleles\ at\ that\ locus}$$

Similarly, the frequency of the a allele is:

$$f_2(a) = \frac{the\ number\ of\ copies\ of\ the\ a\ allele\ in\ the\ population}{the\ total\ number\ of\ copies\ of\ all\ alleles\ at\ that\ locus}$$

In a diploid population (where each individual has two copies of each position), if n represents the total number of individuals, then the total number of copies of all alleles is 2n.

Mathematical example

Assume a population of 200 individuals (ie n = 200) that has two alleles at a particular position: M and m. We count the frequencies of the alleles in the population and observe the following:

80 individuals are homozygous for the M allele (MM genotype).

100 individuals are heterozygous (genotype Mm).

20 individuals are homozygous for the m allele (mm genotype).

The total number of copies of the M allele is:

M = 2·80 (from M) + 1·100 (from Mm) = 160 + 100 = 260

The total number of copies of the m allele is:

m = 2·20 (from m) + 1·100 (from Mm) = 40 + 100 = 140

The total number of alleles in the population is 2·n = 400.

The allele frequencies are:

$$f_1(\mathrm{M}) = \frac{260}{400} = \frac{26}{40} = \frac{13}{20} = 0.65$$

$$f_2(\mathrm{m}) = \frac{140}{400} = \frac{14}{40} = \frac{7}{20} = 0.35$$

Note: another distribution can be assumed:

60 individuals are homozygous for the M allele (MM genotype).

100 individuals are heterozygous (genotype Mm).

40 individuals are homozygous for the m allele (mm genotype).

Calculation of allele frequencies

Number of copies of the M allele:

Each individual M contributes 2 copies of M: 60 ·2=120 copies of the M allele

Each Mm individual contributes 1 copy of M: 100 ·1=100 copies of the M allele

The total number of copies of the M allele is: 120 + 100 = 220

Number of copies of the m allele:

Each individual m contributes 2 copies of m: 40 · 2 = 80 copies of the m allele

Each Mm individual contributes 1 copy of m: 100 ·1=100 copies of the m allele

The total number of copies of the m allele is: 80 + 100 = 180

***Example* 2. Hardy-Weinberg equilibrium**

It is a mathematical theory that describes how allele and genotype frequencies are expected to remain constant in a large, ideal population in the absence of other evolutionary forces such as mutations, migration, natural selection, and non-random mating. This population must meet certain conditions: be large enough, random crossover, absence of natural selection, etc.

Let there be two alleles P and p of a certain genetic locus in a population, (which determine the shape of the wings, P represents large transparent wings and p short transparent wings). Suppose the frequency of the P allele is f_1 of the p allele is f_2. According to the Hardy-Weinberg principle, in a population at equilibrium, the genotype frequencies will be distributed as follows:

- genotype frequency P: ${f_1}^2$
- genotype frequency Pp: 2· $f_1 \cdot f_2$
- genotype frequency p: ${f_2}^2$

This distribution remains constant from generation to generation as long as the population remains in Hardy-Weinberg equilibrium.

Mathematical example

Consider a population of dragonflies in which the frequency of the P allele is f_1= 0.6 and the frequency of the p allele is f_2= 0.4.

Calculation of genotype frequencies:

- genotype frequency P: ${f_1}^2 = 0.36$
- genotype frequency Pp: 2· $f_1 \cdot f_2 = 0.48$
- genotype frequency p: ${f_2}^2 = 0.16$

Thus, in a population in Hardy-Weinberg equilibrium, the distribution of genotypes will be 36% P, 48% Pp, and 16% p.

Extended numerical example:

If the total population of dragonflies is 1000 individuals, then the number of individuals with each genotype will be:

P: 0.36 ·1000 = 360 individuals

Pp: 0.48 · 1000 = 480 individuals

p: 0.16 · 1000 = 160 individuals

Conditions for Hardy-Weinberg equilibrium

For the population to remain in Hardy-Weinberg equilibrium, the following conditions must be met:

- the population must be very large, to avoid the effects of random fluctuations in allele frequencies (genetic drift).
- mating must be random (panmixia).
- there must be no migration (gene flow).
- there must be no mutations that change allele frequencies.
- there must be no natural selection favoring a particular genotype.

Note: this data can be modeled with various population sizes, from random species.

***Example* 3. Natural selection**

It represents the process by which traits that confer an advantage in survival and reproduction become more common in a population.

Mathematical example

In a butterfly population, wing color is determined by two alleles, D (dominant) and d (recessive). The D allele confers a color that provides a selective advantage, and the d allele confers a color that makes the butterflies easier to see for predators.

In the first generation, the allele frequencies are: q_1 for D și q_2 for d; $q_1 + q_2 = 1$.

Genotype frequencies are determined by the relationships:

- f(DD) = q_1^2
- f(Dd) = $2 \cdot q_1 \cdot q_2$
- f(dd) = q_2^2

In a future generation, genotype frequencies are influenced by natural selection as follows:

- Butterflies with genotype D have a survival probability of 1 (100% survive).
- Butterflies with the Dd genotype have a survival probability of 0.9.
- Butterflies with genotype d have a survival probability of 0.5.

Calculate the new allele frequencies q_1 and q_2 by natural selection.

Calculation of genotype frequencies after selection:

We consider that the frequencies of genotypes after selection are given by:

$$f'(DD) = f(DD) \cdot 1 = q_1^2$$

$$f'(Dd) = f(Dd) \cdot 0.9 = 2 \cdot q_1 \cdot q_2 \cdot 0.9 = 1.8 \cdot q_1 \cdot q_2$$

$$f'(dd) = f(dd) \cdot 0.5 = q_2^2 \cdot 0.5$$

Normalization of genotype frequencies:

The sum of the frequencies after the selection must be 1, so we calculate this sum:

$$\text{Total} = f'(DD) + f'(Dd) + f'(dd) = q_1^2 + 1.8 \cdot q_1 \cdot q_2 + q_2^2 \cdot 0.5.$$

$$f''(DD) = \frac{q_1^2}{q_1^2 + 1.8 \cdot q_1 \cdot q_2 + q_2^2 \cdot 0.5}$$

$$f''(Dd) = \frac{1.8 \cdot q_1 \cdot q_2}{q_1^2 + 1.8 \cdot q_1 \cdot q_2 + q_2^2 \cdot 0.5}$$

$$f''(dd) = \frac{q_2^2 \cdot 0.5}{q_1^2 + 1.8 \cdot q_1 \cdot q_2 + q_2^2 \cdot 0.5}$$

Calculation of new allele frequencies q_1 and q_1:

The frequency of the new allele q_1' after natural selection can be calculated as:

$$q_1' = f''(DD) + \frac{1}{2} \cdot f''(Dd) = \frac{q_1^2}{q_1^2 + 1.8 \cdot q_1 \cdot q_2 + q_2^2 \cdot 0.5} + \frac{1}{2} \cdot \frac{1.8 \cdot q_1 \cdot q_2}{q_1^2 + 1.8 \cdot q_1 \cdot q_2 + q_2^2 \cdot 0.5}.$$

The frequency of the new allele q_2' after natural selection can be calculated as:

$$q_2' = 1 - q_1' = 1 - f''(DD) - \frac{1}{2} \cdot f''(Dd) = \frac{q_1^2}{q_1^2 + 1.8 \cdot q_1 \cdot q_2 + q_2^2 \cdot 0.5} - \frac{1}{2} \cdot \frac{1.8 \cdot q_1 \cdot q_2}{q_1^2 + 1.8 \cdot q_1 \cdot q_2 + q_2^2 \cdot 0.5}.$$

After calculating the new frequencies of the alleles q_1' and q_2', we can see how natural selection favored the increase in the frequency of the dominant allele D, at the expense of the allele d.

***Example* 4. Genetic drift**

Mathematical example:

a) The Wright-Fisher model

Consider a population of M diploid individuals, each having two alleles (G and g) at a specific gene locus. Suppose the frequency of allele G in the initial generation is p_0. Genetic drift describes the variation in the frequency of the G allele over successive generations due to random fluctuations.

The frequency of the G allele in generation t+1 is given by: $p_{t+1} = \frac{1}{2 \cdot M} \cdot \sum_{j=1}^{2 \cdot M} X_j$, where X_j is a Bernoulli variable that takes the value 1 if the selected allele is G and 0 if it is g.

This model shows us how random fluctuations can lead to the fixation or loss of an allele over time, within a given population.

b) The Moran model

Consider a population of M individuals, with two alleles, G and g. The Moran model assumes that in each generation one individual is randomly selected for reproduction and another individual is selected for replacement.

The frequency of the G allele after a transition is: $p_{t+1} = p_t + \frac{1}{M} \cdot (X - p_t)$, where X is the Bernoulli variable with the probability p_t of taking the value 1 (choosing allele G for reproduction). This model is useful for understanding the dynamics of small populations, where random events can have a greater impact on allele frequency.

c) The theory of time until fixation

Suppose we have a new allele in a population and we want to calculate the average time until it becomes fixed (reaches frequency 1) or until it is lost (has frequency 0).

We can approximate the average time t to fixation for a population of size M with the initial frequency f_0 of the alleles in the following way:

$$t = -2 \cdot M \cdot (f_0 \ln(f_0) + (1 - f_0) \ln(1 - f_0))$$

The formula can be used to estimate how fast a population is evolving as a function of the initial allele frequency and population size.

d) Model of genetic drift with mutation

Consider a population in which both genetic drift and recurrent mutations occur between two alleles, G and g. Suppose that the probability of a mutation from G to g is μ, and from g to G is ϑ.

The equilibrium frequency of the G allele is given by: $f = \frac{\vartheta}{\mu + \vartheta}$.

The model shows how the balance between mutations and genetic drift can determine the frequency of alleles in a population.

e) Genetic drift in subdivided populations

Consider a species divided into several isolated subpopulations, each of size M, and analyze the effect of genetic drift on genetic variation among subpopulations.

The kinship coefficient F_{dg}, which measures the genetic differentiation between subpopulations, is given by: F_{dg}= $\frac{1}{4 \cdot M \cdot n}$, where n is the rate of migration between subpopulations. This coefficient F_{dg}, is essential for understanding the degree of genetic differentiation between subpopulations due to genetic drift and migration.

***Example* 5.1. Gene Flow (Migration)**

Mathematical Models of Migration and Genetic Evolution

A mathematical application of gene flow is the use of mathematical models to analyze how migration affects the genetic structure of populations. Island models or stepping stone models are often used to describe gene exchange between populations in the form of networks. In these models, migration probabilities and mutation rates are used to calculate the distribution of alleles in various populations and to estimate the time to fixation of a particular allele.

Island Model (Island Model)

In the island model, we imagine a population divided into several isolated subpopulations (islands), each of which has a finite size M. Individuals from each subpopulation can migrate to other subpopulations with a certain probability. We will analyze how gene flow affects the frequency of a particular G allele in these subpopulations.

Defining Parameters:

- $f_j(t)$: frequency of allele G in subpopulation j at time t.
- p: the probability that an individual migrates from one subpopulation to another in each generation.
- $\overline{f(t)}$: mean frequency of the G allele in the total population at time t.

The Recursion Equation

The frequency of the G allele in the next generation (t+1) in a given subpopulation j can be calculated as: $f_j(t+1) = (1-\varepsilon) \cdot f_j(t) + \varepsilon \cdot \overline{f(t)}$, where:

- $f_j(t)$ – its value f_j on time *t*;
- $f_j(t+1)$ – its value f_j on time *t+1*;
- ε – a parameter (usually 0 or 1) that controls how much the new value $f_j(t+1)$ is influenced by the current value $f_j(t)$ in relation to the reference one $\overline{f(t)}$;
- $\overline{f(t)}$ – average reference value at time t.

Observation:

- If ε = 0, then $f_j(t+1)$ = $f_j(t)$, which shows that the value remains unchanged;
- If ε = 1, then $f_j(t+1)$ = $\overline{f(t)}$, which expresses that the value is completely replaced by the reference value;
- If 0< $\varepsilon < 1$, the new value $f_j(t+1)$ represents a combination of the current value and the reference value.
- (1−ε) - represents the fraction of individuals that do not migrate and therefore retain their allele frequency$f_j(t)$ in their current subpopulation. These individuals remain in the subpopulation of origin, keeping the allele frequency constant from one generation to the next.
- ε - represents the fraction of individuals that migrate from other subpopulations and bring with them the average allele frequency $\overline{f(t)}$ from the total population. This contributes to the modification of the allele frequency in the considered subpopulation, based on the influence of alleles from neighboring subpopulations or from the global population.

Mathematical example

Suppose there is a population on an island I with an initial allele frequency $f_I(0) =$ 0,6 and another continental population with a frequency of the allele $f_\varepsilon(0) = 0{,}8$. Migration rate ε = 0.1. We can calculate in this case the allele frequency in the next generation $f_I(1)$=$(1-0{,}1)\cdot 0{,}6 + 0{,}1 \cdot 0{,}8 \ = \ 0{,}62$. Thus, the allele frequency increased due to migration from the continental population.

The two-island model

Consider two islands, P and Q, each having a distinct population with its own frequency, f_P and f_Q respectively. Let ε be the probability that a certain individual from island P migrates to island Q, and ϑ the probability that an individual from island Q migrates to island P. In this case, the new allele frequency on island P after migration can be calculated as:

$$f_P' = (1-\varepsilon)\cdot f_P + \ \vartheta \cdot f_Q.$$

Analogue for island Q:

$$f_Q' = (1-\vartheta)\cdot f_Q + \ \varepsilon \cdot f_P.$$

These calculations can also be specified numerically, for example f_P = 0,6; f_Q = 0,3, $\varepsilon =$ 0,1, $\vartheta = 0{,}05$. Then $f_P' = 0{,}555$ and $f_Q' = 0{,}345$.

***Example* 5.2. Patterns of Gene Flow and Ecological Stability**

Ecological Stability in a Fragmented Landscape

Consider two habitats A_1 and A_1, which have different population sizes, and different survival probabilities a_1 and a_2. They migrate according to the following rule: m_{12} from A_1 to A_2 and m_{21} from A_2 to A_1.

The effective growth rate R_1 and R_2 within the habitats can thus be moderated:

$$R_1 = a_1 + m_{21} - m_{12} \text{ și } R_2 = a_2 + m_{12} - m_{21}.$$

Mathematical example

As a numerical calculation we can have: suppose a_1 = 0,6 and a_2 = 0,4, m_{12} = 0,3 and m_{21} = 0,2.

$$R_1 = 0{,}6 - 0{,}1 = 0{,}\ 5$$
$$R_1 = 0{,}4 + 0{,}1 = 0{,}\ 5.$$

From this we can conclude that both habitats are stable in the presence of migrations and ecological conditions.

***Example* 5.3. Coalescence Theory and Gene Flow**

We can take as an example the estimation of the time to the most recent common ancestor.

In a subdivided population, the coalescence time for two randomly chosen genes from two different subpopulations can be approximated by: $t_{coal} = \frac{2 \cdot m}{r}$ where m is the population size and r is the migration rate.

Mathematical example

If we take a random value m = 2000 and r = 0.02 (2% migrate per generation), then $t_{coal} = \frac{2 \cdot 2000}{0{,}02}$ = 200000, generations (this is the time to the most recent common ancestor for the two genes).

***Example* 5.4. Mathematical Models of Gene Flow in Continuum Space**

As an example we can consider the diffusion of alleles in a continuum space.

Modeling gene flow in a continuous space can be done by the diffusion equation: $\frac{\partial f(x,t)}{\partial t} = D \frac{\partial^2 f(x,t)}{\partial x^2}$, where f is the allele frequency at position t, D is the diffusion coefficient.

Mathematical example

We initially assume that f(x,0) is concentrated at a single point and that D = 0.02 in one minute. At a certain time t = 8, the solution can be: $f(x, 8) = \frac{1}{\sqrt{4\pi D t}} \cdot e^{\frac{-x^2}{4Dt}}$. For x = 0, $f(0, 8) = \frac{1}{\sqrt{4\pi \cdot 0{,}02 \cdot 8}} \cdot e^{\frac{-0^2}{4 \cdot 0{,}02 \cdot t}}$, which shows that the frequency of the allele has dispersed over time in a geographic space.

IV. Conclusions

This brief history of interdisciplinarity as well as these example models provide a deep understanding of evolutionary relationships and the natural history of life. Rigorous mathematical and statistical methods, combined with molecular and morphological data, allow the precise reconstruction of evolutionary trees and the understanding of the evolutionary processes that have shaped biological diversity, as well as the ideal way of coexistence of several species in a natural area, or ways to avoid some epidemics, etc.

The application of mathematics in high school studies is a fundamental pillar in the formation of a rigorous and multidisciplinary thinking, capable of approaching complex problems from a variety of perspectives. Not only does it facilitate deep understanding of abstract concepts, but also expands the ability to transfer these principles to diverse fields such as the exact sciences, economics or even the arts. Through applied mathematics, students develop superior skills in critical analysis, modeling and data interpretation, essential in a world dominated by complexity and interconnectedness. This practical dimension of mathematics not only reinforces theoretical knowledge, but also paves the way for innovation and real-world problem solving, preparing students for the academic and professional challenges of the future. In essence, the application of mathematics transcends the academic sphere, cultivating an analytical and adaptable mindset necessary in an ever-changing society.

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