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PROBLEEMS WITH PARAMETERS AS A VALUABLE TEACHING TOOL OF THE INTELLECTTUAL DEVELOPMENT OF STUDENTS

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Abstract. In the current context of modern education, the emphasis is increasingly shifting from the mechanical accumulation of knowledge to the intellectual development of tje student, to the creative solution of problems. Problems with parameters prezent an important area of didactis of mathematics, which manifests the basic role in the development of logical thinking, cognitive flexibility and creativity of students. This type of problems contributes not only to the consolidation of theoretical knowledge, but also to the formation of skills of analysis, generalization and argumentation. The paper proposes concrete metodical strategies for integrating problems with parameters into the school curriculum. The rezults of the research demonstrate that the systematic use of this teaching tool not only enhances academic performance, but also cultivates superior intellectual capacities, preparing the student for solving complex problems in real life.

Keywords: problems with parameters, problem solving, intellectual education, research activities, teaching tool, intelellectual capabillities.

PROBLEME CU PARAMETRII CA INSTRUMENT DIDACTIC VALOROS AL DEZVOLTĂRII INTELECTUALE A ELEVILOR

Rezumat. În contextul actual al educației moderne, accentul se mută din ce în ce mai mult de la acumularea mecanică a cunoștințelor la dezvoltarea intelectuală a elevului, la rezolvarea creativă a problemelor. Problemele cu parametri reprezintă un domeniu important al didacticii matematicii, care manifestă rolul fundamental în dezvoltarea gândirii logice, a flexibilității cognitive și a creativității elevilor. Acest tip de probleme contribuie nu numai la consolidarea cunoștințelor teoretice, ci și la formarea abilităților de analiză, generalizare și argumentare. Lucrarea propune strategii metodice concrete pentru integrarea problemelor cu parametri în programa școlară. Rezultatele cercetării demonstrează că utilizarea sistematică a acestui instrument didactic nu numai că îmbunătățește performanța academică, dar și cultivă capacități intelectuale superioare, pregătind elevul pentru rezolvarea problemelor complexe în viața reală.

Cuvinte cheie: probleme cu parametri, rezolvarea problemelor, educație intelectuală, activități de cercetare, instrument didactic, capacități intelectuale.

Parametric problems are one of the most powerful teaching tools in developing students' intellectual capacities, as they develop logical thinking, flexibility, the ability to analyze and generalize. Their importance is underlined by the fact that they contribute to the deep understanding of the relationships between quantities and to the modeling of real phenomena. In this sense, parameter problems are not just abstract exercises, but tools of mathematical inquiry.

A parameter problem is the mathematical problem in which one or more independent variable sizable quantities, and the solution depends on their values. The parameter can influence the existence, uniqueness, or shape of the solution.

A parameter problem is the mathematical problem in which one or more independent variable sizable quantities, and the solution depends on their values. The parameter can influence the existence, uniqueness, or shape of the solution.The parameter represents an independent variable whose value in a problem is considered a task, a fixed or arbitrary real number, or a number belonging to a predetermined set. From a didactic point of view, an equation with parameters is interpreted as a **family of equations**, obtained by fixing the parameter to particular values. This approach highlights the fact that the parameter is not an ordinary unknown, but a quantity that influences the structure of the solutions [1, 2].

Research hypothesis: *Problems with parameters are not only a difficult mathematical content, but a genuine didactic tool for the development of flexible thinking, for the formation of mathematical thinking.*

The purpose of this work *is to highlight the formative value of problems with parameters in the intellectual development of students.*

The intellectual development of the student by solving problems with parameters requires the establishment of didactic objectives aimed at this process. These objectives must be formulated in accordance with the higher cognitive levels highlighted by Benjamin S. Bloom, such as analysis, synthesis and evaluation.

Didactic objectives achieved in mathematics classes are:

1. ***Cognitive objectives***
   - Understanding the concept of parameter and differentiating it from the unknown;
   - Identifying the conditions for the existence of solutions;
   - Recognizing the dependence of solutions on parameter values;
   - Applying solution methods in variable contexts.

These objectives contribute to the *development of analytical and logical* thinking by requiring the decomposition of the problem into distinct cases.

2. ***Operational objectives***

   The student will be able to:
   - Identify and treat different cases of the parameter;
   - Solve each case correctly;
   - Formulate general conclusions;
   - Check the validity of the solutions [4].

These objectives develop *the rigor and discipline of thinking.*

3. ***Attitudinal objectives***
   - Developing interest in mathematical investigation;
   - Cultivating perseverance in solving problems;
   - Forming confidence in one's own abilities.
4. ***Formative objectives***, that reflect the idea of L. Geoge Polya, according to which problem solving develops thinking strategies and intellectual autonomy [7].

Problems with parameters are essential in the development of intellectual capacities through their *formative role* presented in table 1.

Table 1. The formative role of problems with parameters in the development of intellectual capacities of students

Intellectual ability	The role of problems with parameters	Formative effect on the student
Abstract thinking	Working with variables and parameters instead of fixed numerical values	Students go from concrete to abstract and understand general structures
Ability to generalize	Analyzing solutions according to different parameter values	Formulates valid conclusions for this type of problems
Logical reasoning	Solving by cases and establishing existence conditions	Develops deductive thinking and correct argumentation
Critical thinking	Comparing solution methods and choosing strategy	Develops autonomy in decision-making
Creativity	Ability to approach the problem with more than one method	Stimulates original solutions and intellectual flexibility
Analysis and synthesis	Interpreting data and formulating the overall result	Develops the ability to organize and structure information
Knowledge transfer	Applying concepts to new, varied situations	Prepares students for complex and interdisciplinary problems

The table highlights the fact that problems with parameters have a complex formative role, simultaneously contributing to the development of several dimensions of students' thinking. The teacher creates an observation grid and takes into account the achievement of the proposed objectives.

Applied methods

Methods of teaching problems with parameters and their formative role:

1. *The heuristic method* – guiding students through dialogue to discover the relationship. between parameters and and solutions (students learn to formulate hypotheses and argue).
2. *Problematization* – creation of situations - problem, generation of cognitive conflict (students become active and look for solutions) [5].
3. *Discovery learning* – students identify general rules from particular cases (deep and lasting understanding).
4. *Exercise method* – repeated solving of various problems with parameters (training of working skills).

5. *Mathematical modeling* – building models that include parameters (students understand relationships between quantities).
6. *Cooperative learning* – group activity for problem solving (exchange of ideas and multiple solutions) [6].

Didactic strategies used in solving problems with parameters are:

1. *According to the logic of thinking*
   - Inductive strategies – apply to the analysis of several values of the parameter to formulate a general conclusion;
   - Deductive strategies – application of general rules for determining solutions according to parameters [4].
2. *By degree of direction*
   - Heuristic strategies – based on discovery and investigation, students discover relationships between parameters and solutions;
   - Participatory strategies – group problem solving is applied with debates on the solutions [6].
3. *According to the way of organization*
   - Individual strategies – independent solving activity;
   - Group strategies – collaborative activity with the application of team solutions.
4. *According to the didactic purpose*
   - Training strategies – training and development of logical thinking and generalization capabilities;
   - Evaluation strategies – problems with parameters in evaluation tests [4, 6].

Results and discussions

***Example* 1**. For what values of parameter t equations

$$2x^2-(3t+2)x+12=0 \quad \text{and} \quad 4x^2-(9t-2)x+36=0$$

do they have a common root?

Solution. We know that if for a value of the parameter *t*, both equations have the common root x_1, then the pair $(x_1,\ t_1)$ is the solution of the system formed by these two equations. So, we need to determine the solution of the system formed by these two equations:

$$\begin{cases}2x^2-(3t+2)x+12=0,\\4x^2-(9t-2)x+36=0.\end{cases}$$

To solve such a system, we make up the equation in which the free term is missing. For this purpose we multiply the first equation by (–3) and add with the second equation. As a result we get the equation:

$$-2x^2+8x=0.$$

The roots of this equation are $x_1 = 0, x_2 = 4$. The first value cannot be a solution of the system, because in that case the first equation turns into $12 = 0$. If we substitute the value $x = 4$ in the first equation, we get:
$32 - 4(3t + 2) + 12 = 0,\ \ -12t + 36 = 0$ and $t = 3$. If we replace the values $x = 4$ and $t = 3$ in the second equation we obtain a true identity.
Response: $t = 3$

Solving this problem with parameters develops several important intellectual capacities of the student because the method of solving requires the analysis, synthesis, argumentation and integral application of algebraic knowledge:

- *Logical thinking* is developed by the fact that it is necessary to establish correct relationships between equations in order to follow a logical reasoning to determine the parameter; the development of the ordering of the stages of solving takes place; mathematical argumentation is developed.
- *The ability to analyze and synthesize* is developed by analyzing each equation; identifying the common root; the student learns to break down the problem into solution steps and then to synthesize the results obtained.
- *The flexibility of thinking* is manifested by the way of choosing the method of elimination; substitution; comparing the relationships obtained. All these elements develop intellectual adaptability and changing work strategies.
- *Abstract thinking* develops due to the fact that the parameter t represents a variable value, and the student operates with general relationships, not with concrete numbers; this is how abstraction develops; generalization; symbolic operation.
- *The ability to investigate involves* investigating the conditions in which two equations have the property of having a common root; it follows with the formulation of the hypotheses, the verification of the results and their interpretation.
- *Attention and precision* are manifested by carefully performing calculations and checking the results obtained, intellectual discipline is developed.

Mathematical problems with parameters of this type represent an effective tool for the intellectual development of the student.

***Example* 2**. *To find out for which values a of the parameter of the equation*

$$\sqrt{x^2 + 6x + 5} = \sqrt{a - 6x}$$

has exactly one negative root.

Solution. This example was proposed to the students after studying the topic 'Solving irrational equations'. The students replaced the given equation with the equivalent system.

$$\begin{cases} x \in (-\infty; -5] \cup [-1; +\infty), \\ \quad x^2 + 12x + 5 - a = 0. \end{cases}$$

They ask the question about the new problem (find out the values of the parameter a for which the equation has exactly a negative root on the intervals).

On the teacher's side, the heuristic questions follow, which guide the students' thinking and research activity:

1) Has the quality of the equation changed as a result of the equivalent transformation? (Yes, the irrational equation has been replaced by the square equation)
2) Formulate the question about the new problem, to find the values of the parameter a for which the equation has exactly one negative root on the intervals $(-\infty;\ -5]\ and\ [-1;+\infty)$.
3) How many unknowns are in the given equation? What is the solution to the problem in this case?
4) Where the ordered pair of numbers can be represented $(x,\ a)$? What should be done in such a case? (In a rectangular coordinate system xOa one variable must be expressed by the other).
5) Do this (Students construct the parabola $a = x^2 + 12x + 5$ and indicates the geometric location of points of the coordinate plane corresponding to the inequalities)
6) Students analytically determine the parameter values: $a = -31,\ a(-5) < a < a(-1)\ , a \geq a(0)$ and get the answer.

Response: $\{-31;\ (-30;-6); [5;+\infty]\}$

Problems of this type develop the following intellectual capacities:

- *Logical thinking and deductive reasoning*, the student must follow the following steps: establishing the conditions of existence, transforming the equation, solving and analyzing radicals. Observance of these steps leads to the development of the ability to argue and the correct formulation of the answer.
- *The ability to analyze and interpret* is developed during the analysis of the solutions found, in this way the development of analytical thinking takes place.
- *Cognitive flexibility* is developed by passing during the solution from an irrational equation to an algebraic equation and then to the study of the sign of the roots, thus adapting the solution strategies.
- *The ability to generalize* is developed through the simultaneous analysis of a infinities of cases.

During the solution of the problem with the modification of the parameter, the mathematical exploration and investigation takes place.

***Example* 3**. In the acute-angled triangle ABC, where *m*(∠BAC)=α^0 is the height AH. To find out the values. To find out the values of the parameter α for which the height AH will be equal to 1, if AB=2, AC=3. (Figure 1)

Solution:

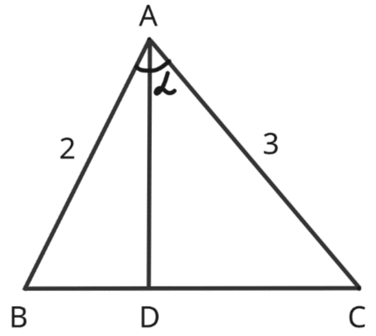

Figure 1. Graphical representation of the problem 3

Since the triangle ABC is acute-angled, then $\alpha \in (0^0; 90^0)$ from the theorem of cosines we find the side BC: $BC^2=AB^2+AC^2-2AB\cdot AC\cdot\cos\alpha$ and we get $BC=\sqrt{13-12\cos\alpha}$.
We will calculate the area of triangle ABC by 2 methods:

1) $A=\frac{1}{2}AB\cdot AC\cdot\sin\alpha \Leftrightarrow A=3\sin\alpha$;
2) $A=\frac{1}{2}AD\cdot BC \Leftrightarrow A=\frac{1}{2}\sqrt{13-12\cos\alpha}$.

Equating the right sides for the area of the triangle, we get:

$$3\sin\alpha=\frac{1}{2}\sqrt{13-12\cos\alpha}.$$

Because $\alpha \in (0^0; 90^0)$, follows $\sin\alpha>0$. We square both sides of the obtained equation and after bringing it to the canonical form, we get:

$$36\cos^2\alpha-12\cos\alpha-23=0.$$

The roots of this equation are:

$$\cos\alpha=\frac{1\pm2\sqrt{6}}{6}.$$

Considering that $\alpha \in (0^0; 90^0)$, follows $\cos\alpha>0$. In that case the value $\cos\alpha=\frac{1-2\sqrt{6}}{6}$ does not satisfy the condition of the problem. As a result we get:

$$\cos\alpha=\frac{1+2\sqrt{6}}{6},\ \alpha=\arccos\left(\frac{1+2\sqrt{6}}{6}\right).$$

Response: $\alpha=\arccos\left(\frac{1+2\sqrt{6}}{6}\right)$.

This problem develops several components of fluid intelligence. It thus capitalizes on higher cognitive processes such as: abstraction, mathematical modeling, cognitive flexibility and critical thinking. The student becomes obliged to establish relationships between geometric and algebraic concepts, to investigate and logically validate the obtained solution.

In the context of solving problems with parameters, the assessment of students' intellectual abilities can be carried out both by tests and by *observation grids of the solving activity*, which analyze how the student formulates hypotheses, analyzes the conditions of the problem and argues the results obtained. The level of development of these skills can be assessed on several levels: guided by the teacher, partially independent, completely independent. Both at the initial stage and at the final stage of the pedagogical experiment, tests were carried out with the aim of establishing the initial and final level of development of the students' intellectual capacities. The initial test includes four problems, which aim to assess intellectual capacities:

- Identifying the relationships between coefficients, logical analysis, algorithmic thinking (problem 1);
- Applying the discriminant, analytical thinking, formulating mathematical conclusions (problem 2);
- Structural analysis of systems, logical thinking, deductive reasoning (problem 3);
- The ability of mathematical analysis, logical thinking, argumentation and generalization (problem 4).

The final evaluation test aimed to assess the final level of abilities:

- Interpretation of partial cases, logical analysis, deductive thinking (problem 1);
- Use of the discriminant, analytical thinking, mathematical modeling (problem 2);
- Geometric interpretation, mathematical analysis, functional thinking (problem 3);
- Analysis of the conditions of existence, critical thinking (problem 4). The tests were the same for both groups of students (EG and CG). The frequency and distribution of pre-test grades for the experimental group (EG) and the control group (CG) are shown in Table 2.

Table 2. Frequency and distribution of grades in EG and CG at the initial stage of the experiment

Group	Marks	10	9	8	7	6	5	Total
EG	No. of students	7	8	12	9	11	3	50
	/%	14	16	24	18	22	6	100
CG	No. of students	5	7	12	14	6	1	45
	/%	11,1	15,6	26,7	31,1	13,3	2,2	100

The distribution of grades obtained at the initial stage of the experiment shows that the majority of students in both groups obtained results located in the range of 7-9. The majority of students obtained average and good grades, which confirms that EG and CG have a comparable initial level of preparation. The distribution and frequency of students' post-test grades in both CG and CG are shown in Table 3.

Table 3. Frequency and distribution of grades of EG and CG students at the final stage of the experiment

Group	Marks	10	9	8	7	6	5	Total
EG	No. of students	18	19	5	4	3	1	50
	/ %	36	38	10	8	6	2	100
CG	No. of students	8	9	12	8	7	1	45
	/ %	17,8	20,0	26,7	17,8	15,6	2,2	100

In the experimental group, most students obtained very good grades: grade 9-38% and grade 10 – 36%, which shows clear progress without a pre-test. In the control group, the distribution is more balanced, with most students between 8 and 9, but the percentage of maximum marks (10) is much lower (17.8). The number of students with lower grades (5 and 6) decreased in both groups, but more noticeably in EG. Comparing the results from the post-test with the pre-test, a significant increase in the performance of students in the experimental group is observed, which suggests the effectiveness of activities related to solving problems with parameters. The control group shows a more moderate increase, which confirms that the experimental method had a positive impact on the development of students' intellectual capacities.

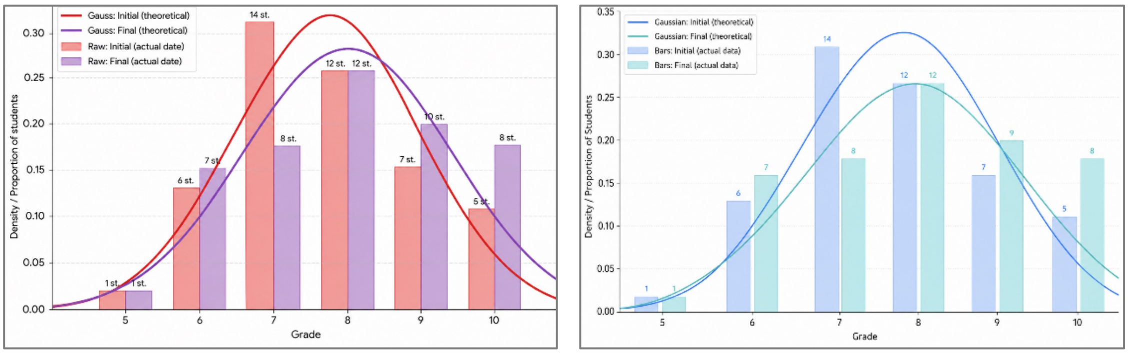

Figure 2. Comparative analysis of Gaussian distribution and grade frequency in EG and CG (at the initial and final stages of the experiment)

In the experimental group the average grades increased from 7.64 to 8.84. The shape of the Gaussian curve at the final stage is strongly shifted to the right. Its peak is in the area of high notes. The share of grades 9 and 10 at the final stage belongs to 37 out of 50 students, while the small grades 5, 6, 7 have almost disappeared. group homogeneity increased from 1.50 to 1.28. in the control group the evolution is minimal. The mean increased insignificantly from 7.73 to 8.00. the shape of the Gauss curve at the final stage practically keeps the same position and widening only towards the extremity, the distribution remains bell-shaped, where grades 7 and 8 remain predominant, the standard deviation increased from 1.27 to 1.40. only at the entrance the pedagogical experiment in EG was applied and worked successfully. Solving problems with parameters develops a valuable set of intellectual capacities, as students no longer work only with fixed values, but must think broadly and flexibly. When assessing intellectual abilities, some skills are more visible and easier to measure, and others are more subtle or difficult to assess. Easier to evaluate are:

- *logical and deductive reasoning* (the steps to solve the problem are clear, the answer can be checked);
- *analysis and generalization* (easily evaluated by the correct answer);

➢ *application of knowledge* (it is easy to check if the student has mastered certain knowledge).

More difficult to evaluate are: *creativity, flexibility of thinking, abstract thinking, transfer of knowledge, autonomy in learning* (these intellectual capacities to be graded require observation during the solving process). The distribution of EG students by levels of developed abilities is shown in Figure 3, and in CG is shown in Figure 4.

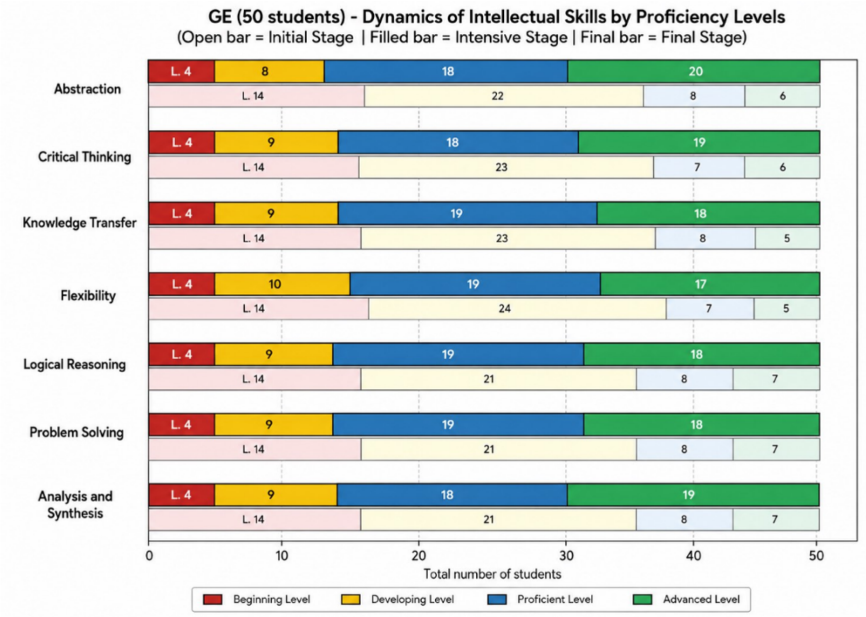

Figure 3. Evolution of the levels of intellectual capacities developed in the experimental group (initial and final stage)

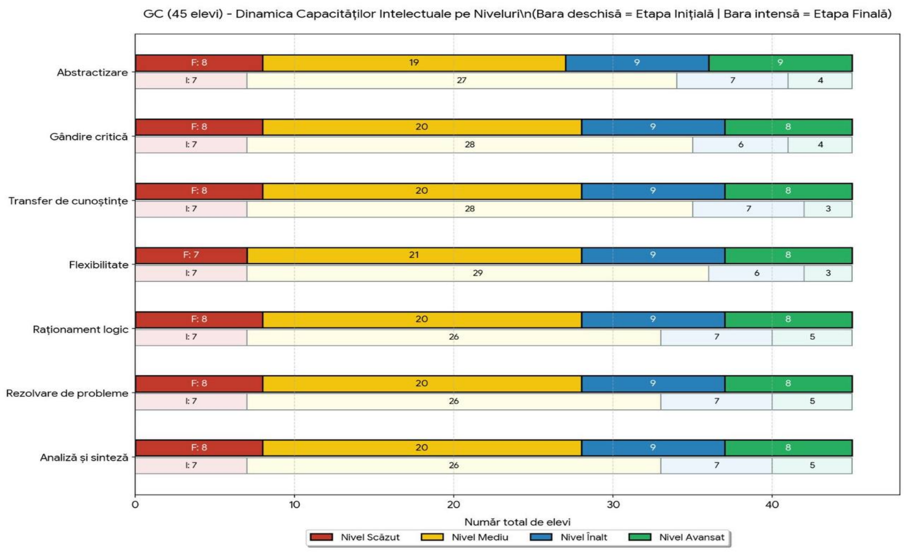

Figure 4. Evolution of the levels of intellectual capacities developed in the control group (initial and final stage)

These horizontal, segmental diagrams show how the structure of the groups changed for each of the 7 intellectual capacities following the application of the expected didactic experiment.

Conclusions

Problems with parameters represent an effective and valuable didactic tool in the intellectual development of students, because they require from the student the use of logical thinking, the ability to analyze, generalize and cognitive transfer. The open learning situation stimulates flexibility and the formation and development of logical reasoning for solving problems with parameters. Grade 10 research demonstrates the important role and formative potential of parameter problem solving in mathematics. The results obtained and presented in figure 2 and figure 3 no longer refer only to general marks, but also to 7 specific intellectual capacities, each evaluated on 4 levels: Advanced, High, Medium and Low, investigated at the initial and final stage of the experiment. Following the implementation of the experimental didactic approach in the experimental group, a qualitative restructuring of all intellectual capacities is clearly observed with massive migrations of students from vulnerable areas to high and advanced level areas. In the control group there was a minor and insignificant migration from the levels. The integration of problems with parameters in the didactic process can be *a valuable direction for the modernization of the educational process according to the requirements of modern education.*

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