

MODERN TECHNIQUES FOR SOLVING PARAMETER PROBLEMS WITH DIDACTIC APPLICATIONS IN THE DEVELOPMENT OF HIGHER COGNITIVE ABILITIES

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Abstract. The article examines the optimization of solving mathematical problems with parameters through modern heuristic strategies aimed at developing higher-order cognitive skills. The theoretical-methodological research draws on ERIC-indexed literature and recent studies on heuristics, complex problem-solving, and mathematical thinking. It analyses techniques such as systematic parameter variation, specialization and generalization, case analysis, multiple representations, mathematical modelling, heuristic dialogue, and metacognitive reflection. The main contribution is a methodological model integrating these techniques into the problem-solving process and highlighting the cognitive mechanisms involved at each stage. The proposed approach promotes critical thinking, cognitive flexibility, and the transfer of problem-solving strategies to new contexts.

Keywords: parameter problems, problem solving, research activities, intellectual abilities, didactic process, metacognition, variational thinking, 5D method, heuristic approaches.

TEHNICI MODERNE DE REZOLVARE A PROBLEMELOR CU PARAMETRI CU APLICAȚII DIDACTICE ÎN DEZVOLTAREA ABILITĂȚILOR COGNITIVE SUPERIOARE

Rezumat. În articol se examinează optimizarea rezolvării problemelor matematice cu parametri prin strategii euristice moderne, orientate spre dezvoltarea abilităților cognitive de nivel superior. Cercetarea teoretico-metodologică se bazează pe literatura indexată în baza de date ERIC și pe studii recente privind euristica, rezolvarea problemelor complexe și dezvoltarea gândirii matematice. Sunt analizate tehnici precum variația sistematică a parametrilor, particularizarea și generalizarea, analiza pe cazuri, utilizarea reprezentărilor multiple, modelarea matematică, dialogul euristic și reflecția metacognitivă. Contribuția principală constă în elaborarea unui model metodologic care integrează aceste tehnici în procesul de rezolvare și evidențiază mecanismele cognitive implicate în fiecare etapă. Abordarea propusă stimulează gândirea critică, flexibilitatea cognitivă și transferul strategiilor de rezolvare în contexte noi.

Cuvinte cheie: probleme cu parametri, rezolvarea problemelor, activități de cercetare, abilități intelectuale, proces didactic, metacogniție, gândire variațională, metoda 5D, abordări euristice.

Introduction

Techniques for solving problems with parameters represent the set of cognitive and mathematical procedures that are used to investigate the influence of the parameter on the structure of the problem, to build a mathematical model so that we can identify the solution to the problem through reasoning, generalization and optimization. *Heuristic techniques* for solving parameter problems are the cognitive and mathematical processes

that support the systematic exploration of parameter variation, the construction and validation of mathematical methods, and the formulation of reasoned and generalizable solutions. The topicality of this topic lies in the heuristic approach. Scholars in the current scientific world in terms of researching parameter-dependent problems is quite dynamic and can form groups according to approaches to this topic:

- Approach through pure mathematics and optimization algorithms (create new formulas and theorems to reduce calculation time when a system has too many parameters);
- Jan S, advocates Reduced Basis Methods for parameter and time-dependent problems, he demonstrates how one can approve exact solutions of parameter equations without consuming too much computation;
- Didactic and heuristic approach;
- Approach by intelligence. Vivek Shenov develops innovative mathematical models to solve some scientific problems where the equations critically depend on hidden parameters.

Recent studies (2025-2026) conducted in the European Academic Area have established that there is an increased interest in researching how *parameters develop critical thinking*. Studies published by the ERIC (Education Resources Information Center) platform analyze *heuristic approaches and cognitive psychology* in solving parameter problems. Likewise, emphasis was placed *on the importance of correlating algebraic and geometric representations*. Heuristic methods proposed by the ERIC platform in the development of higher cognitive capacities by solving problems with parameters are:

- The 5D method;
- Change of semiotic register (Duval);
- Variational analysis (conceptual variation).

In modern didactics of STEM subjects, managing the logical bifurcations generated by the presence of an abstract parameter represents a major cognitive challenge for pupils and students. The *method of the 5 Ds* (Definition, Decomposition, Diagramming, Derivation, Decision) is presented as a metacognitive tool, which functions as a system of successive filters in the step-by-step solution of the problem with parameters.

The present work proposes a structured analytical/methodical approach, which seeks to apply some techniques for solving problems with parameters as a mental process of strategy that simplifies a difficult situation, traces a clear path to a correct solution, a working tool that will help to see the real cause of an obstacle in solving.

The aim of the work consists in the theoretical substantiation and methodological validation of some modern, heuristic techniques for solving problems with parameters, with an emphasis on the 5D Heuristic Model and Variational, Conceptual Analysis,

demonstrating their effectiveness as didactic tools in the development of higher cognitive capacities.

To achieve this goal, the following objectives were proposed:

- Investigating the literature indicated in international databases such as ERIC;
- Detailing the cognitive mechanism and techniques applied in the model heuristic 5D
- Designing and carrying out case studies with the application of heuristic models
- Evaluation of the didactic impact achieved by implementing these techniques in the teaching process.

Usually the following problem with parameters is solved as follows:

Example 1. The sides of the triangle are located on the coordinate axes and on the tangent of the graph of the function $y = x^2 + 2x + 1$ at the point whose abscissa satisfies the condition $-\frac{1}{2} \leq a \leq 0$. Find the value of a for which the area of the triangle will be the largest.

Solution. The equation of the tangent, taken to the graph of the function $y = f(x)$ at the point with the abscissa $x_0 = a$, has the shape $y - f(a) = f'(a)(x - a)$.

From the condition of the problem $f(x) = x^2 + 2x + 1 = (x + 1)^2$, $f'(a) = 2(a + 1)$, and the coordinates $A(x_0; 0)$ and $B(0; y_0)$ are determined from the systems:

$$\begin{aligned} & \begin{cases} x = 0 \\ y = (a + 1)^2 + 2(a + 1)(x - a) \end{cases} \cup \begin{cases} y = 0 \\ y = (a + 1)^2 + 2(a + 1)(x - a) \end{cases} \\ & \Leftrightarrow \begin{cases} x = 0 \\ y = (a + 1)(2x + 1 - a) \end{cases} \cup \begin{cases} y = 0 \\ y = (a + 1)(2x + 1 - a) \end{cases} \\ & \Leftrightarrow \begin{cases} x = 0 \\ y_0 = 1 - a^2 \end{cases} \cup \begin{cases} y = 0 \\ x_0 = \frac{a - 1}{2} \end{cases}. \end{aligned}$$

Obviously, the obtained triangle is right-angled, but then the area of the triangle will be equal to the semi-product of the legs.

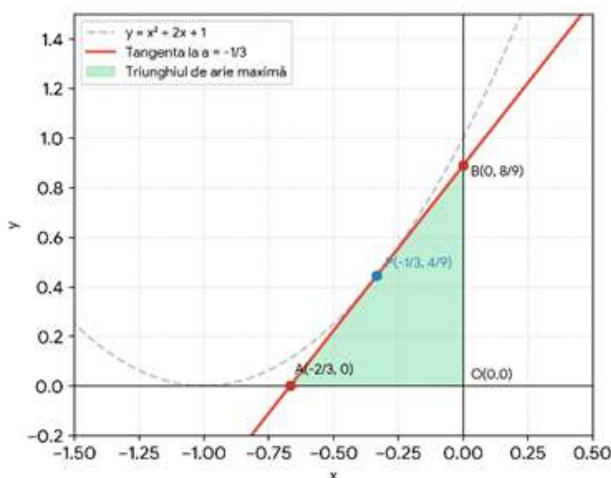


Figure 1. Graphical representation of the parabola, the tangent and the maximum triangle formed

Therefore,

$$\begin{aligned} S_{\Delta AOB} &= \frac{1}{2} |AO| \cdot |OB| = \frac{1}{2} \left| 1 - a^2 \right| \cdot \left| \frac{a-1}{2} \right| = \\ &= \frac{1}{4} |(1 - a^2)(a - 1)| = \frac{1}{4} (1 - a^2)(a - 1). \end{aligned}$$

$S = \frac{1}{4}(a^3 - a^2 - a + 1)$, because for $a \in \left[-\frac{1}{2}; 0\right)$, $|1 - a^2| = 1 - a^2$, $|a - 1| = 1 - a$. Let's determine the value $a \in \left[-\frac{1}{2}; 0\right)$, for which the function $S(a)$ has the maximum value.

$$S'(a) = \frac{1}{4}(3a^2 - 2a - 1) = 0 \Leftrightarrow a = -\frac{1}{3} \cup a = 1.$$

Given that $-\frac{1}{3} \in \left[-\frac{1}{2}; 0\right)$, $1 \notin \left[-\frac{1}{2}; 0\right)$ and that derivative of the function $S(a)$, that is $S'(a)$ for $a = -\frac{1}{3}$ changes its sign from plus to minus, we conclude

$$\max_{a \in \left[-\frac{1}{2}; 0\right)} S(a) = S\left(-\frac{1}{3}\right) = 8/27.$$

Response: $a = -\frac{1}{3}$.

In the specialized literature dedicated to mathematical didactics according to ERIC research, the 5 D method presents an algorithm that accompanies the student in solving the problem. Each D presents a mandatory transformation step of an abstract analysis to solve parameter problems. In the case of solving the previous problem, we propose the methodology of solving the problem with parameters through the 5D method.

Table 1. The structural dynamics of the application of the 5 D Heuristic Model in the cognitive development of students (example 1)

Stages 5D	Heuristic Activity	Mathematical solution
1. Definition	Parameter Isolation	Identifying the restrictive range of the parameter: $a \in (-1/2, 0)$ and the fixed form of the parabola $y=(x+1)^2$.
2. Decomposition	Structural segmentation: decomposing the problem into independent algebraic modules	Calculating the slope of the line by derivation: $m=2(a+1)$ and assembling the analytical equation of the moving tangent: $y=2(a+1)x+(1-a^2)$.
3. Degramming	Translating the equations into a dynamic spatial representation	determining the intersections with the axes as absolute lengths of the legs: $OA=1-a/2$ și $OB=1-a^2$.
4. Derivation	Extreme modeling: synthesizing the geometric components into a single equation –goal and enabling the differential apparatus for optimization	Formulating the area function $S(a)=1/4(1-a)^2(1+a)$, calculating the derivative $S'(a)$ and identifying the critical roots: $a_1=1$ and $a_2=-1/3$
5. The decision	Filtering and final synthesis	Eliminating the parasitic value $a=1$, validating $a=-1/3$ and calculating the maximum area: $S_{\max}=8/27$.

- D_1 . *The definition*-developed cognitive component is critical thinking and self-regulation: the ability to set early logical information processing filters;
- D_2 . *Decomposition*-develops critical analysis and logical decomposition: the ability to reduce complexity by isolating variables;
- D_3 . *Diagramming* develops spatial-visual thinking;
- D_4 . *Derivation* develops formal-operational thinking: the abstract application of high-level mathematical algorithms on functional models;
- D_5 . *The decision* develops critical evaluation and metacognition: the ability to make decisions, to formulate conclusions.

Several heuristic techniques were used in this problem:

1. *Mathematical modeling*: The problem condition was transformed into a mathematical model. The equation of the tangent to the parabola $y = x^2 + 2x + 1$.
2. *Introduction of a parameter*: The abscissa of the point of tangency was denoted by a and all quantities (tangent equation, axis intersections, area) were expressed according to this parameter.
3. *Decomposing the problem into simpler steps*: Determining the tangent, finding the intersections with the axes, expressing the area of the triangle, determining the maximum area.
4. *Reduction to an optimization problem*: After obtaining the area formula $A(a)$, the geometric problem turned into a problem of mathematical analysis: finding the maximum value of a function.
5. *Using derivatives*: Derived the area function, determined the critical points, and compared the values of the function at the critical point and at the ends of the interval to identify the maximum.
6. *Checking the imposed conditions*: The restriction in the statement was taken into account: $-1/2 \leq a \leq 0$, eliminating the solutions that do not belong to the interval.

The heuristic model for solving the problem with parameters by method 5D

Example 2. The function is given

$$g(x) = 7^x + \frac{49}{7^x} = 7^x + 7^{2-x}.$$

Determine the values of the parameter a for which the function $f(x) = g(x + a)$ is even.

Solution. We have:

$$g(x) = 7^x + \frac{49}{7^x} = 7^x + 7^{2-x} \text{ and } f(x) = g(x + a).$$

D_1 - Field of definition: Since $7^x > 0$ for any $x \in \mathbb{R}$, it follows: $D_g = D_f = \mathbb{R}$.

D_2 - Derivative: Calculate the derivative of g : $g'(x) = 7^x \cdot \ln 7 - 7^{2-x} \cdot \ln 7$. So:

$$g'(x) = \ln 7 (7^x - 7^{2-x}).$$

To find the critical point, we equate the derivative with zero:

$$\ln 7(7^x - 7^{2-x}) = 0 \Leftrightarrow 7^x = 7^{2-x} \Leftrightarrow x = 2 - x \Leftrightarrow x = 1.$$

D₃ – We determine the variation: How $\ln 7 > 0$:

- If $x < 1$, then $g'(x) < 0$, so g decreases;
- If $x > 1$, then $g'(x) > 0$, so g increases.

By therefore, g has a minimum in $x = 1$.

Graphically, the function g is symmetric to the right:

$$x = 1.$$

D₄ – We analyze the function:

$$f(x) = g(x + a).$$

Replacing x by $x + a$, the axis of symmetry changes:

$$x + a = 1.$$

So, the graph of f has the axis of symmetry:

$$x + a = 1.$$

For f to be **even**, the graph must be symmetrical to of the Oy axis.

Therefore: $1 - a = 0$.

D₅ – Determination of the parameter: $a = 1$.

Answer: $a \in \{1\}$.

Check:

$$f(x) = g(x + 1) = 7^{x+1} + 7^{1-x},$$

and

$$f(-x) = 7^{1-x} + 7^{x+1} = f(x).$$

So, the function is indeed *even*.

The technology of solving parameter problems by the method of Variational Analysis

The method of variational analysis is not a universal method, although it is very effective for wide classes of parameter problems. It may be suitable to be applied to: equations and inequalities with parameters and their systems; problems where the functions involved are continuous and can be studied by monotonicity, derivatives or convexity; problems in which the number of solutions is required according to the parameter; problems that can be interpreted geometrically.

The method of variational analysis consists in the systematic study of the changes of the solutions according to the variation of the parameter. It is based on the identification of the critical values of the parameter, which separate its domain into intervals where the number and nature of the solutions remain invariant. The analysis of each interval, together with the examination of the critical values, allows the complete description of the solutions for all values of the parameter. The theoretical justification of the method is

based on the properties of continuity, monotonicity and on the functional relationships between the unknown and the parameter.

How to solve example 2 using the Variational Analysis method

Step 1. *Expression of the shifted function.* We have: $f(x) = g(x + a) = 7^{x+a} + 7^{2-a-x}$.

Step 2. *Variational analysis of the function g .* Calculate the derivative of the function g :

$$g'(x) = 7^x \cdot \ln 7 - 7^{2-x} \cdot \ln 7 = \ln 7(7^x - 7^{2-x}).$$

The stationary points are obtained from the condition:

$$g'(x) = 0 \Leftrightarrow 7^x = 7^{2-x} \Leftrightarrow x = 2 - x \Leftrightarrow x = 1$$

Study of the sign of the derivative:

- If $x < 1$, then $7^x < 7^{2-x}$, so $g'(x) < 0$;
- If $x > 1$, then $7^x > 7^{2-x}$, so $g'(x) > 0$.

It turns out that:

- g is strictly decreasing on $(-\infty, 1)$;
- g is strictly increasing on $(1, +\infty)$.

Therefore, the graph of the function is symmetric about the vertical line $x = 1$.

Since $g(1+t) = 7^{1+t} + 7^{1-t} = g(1-t)$, $\forall t \in \mathbb{R}$.

Thus, the axis of symmetry g is $x = 1$.

Step 3. *Effect of translation.* The function $f(x) = g(x + a)$ is obtained by translating the graph of the function g by a units on the Ox .

The axis of symmetry of the function f becomes

$$x = 1 - a.$$

For f to be **even**, the axis of symmetry must coincide with the Oy axis, i.e. $x = 0$.

Therefore,

$$1 - a = 0, \text{ from which it follows that } a = 1.$$

Conclusion: the only value of the parameter for which the function $g(x + a)$ is **even** and $a = 1$.

From a didactic and methodical point of view it is appropriate to simultaneously highlight the method, the heuristic techniques applied and their impact on the development of higher cognitive abilities in pupils and students (Table 2).

**Table 2. Comparative analysis of heuristic techniques
and cognitive abilities developed**

Techniques 5D heuristics	Mathematical manifestation in the given problem	Higher cognitive abilities developed
Defining the even function	Reducing the problem to the condition $f(-x) = f(x)$	Analysis, understanding
Particularizing the definition	Adapting the definition of the even function to the function $f(x) = g(x + a)$	Application, analysis, transfer
Equivalent transformations	Rewriting $\frac{49}{7^x} = 7^{2-x}$ and making the transformations	Application, analysis

Introducing an auxiliary notation (substitution $U = 7^x$)	Simplifying the equation and reducing it to a more accessible form.	Mathematical modeling, abstraction
Verifying the solution	Confirming that for $a = 1$, the function is even.	Evaluation, self-evaluation
Variational analysis method	Mathematical manifestation in solving the given problem	Higher cognitive abilities developed
Study of derivative and monotonicity	Determining intervals of monotonicity and the minimum point	Analysis, application
Identifying the axis of symmetry	Observing the symmetry of the function with respect to the line $x = 1$.	Analysis, interpretation
Geometric interpretation of translation	Establishing the fact that the axis of symmetry of the function $g(x + a)$ is $x = 1 - a$	Spatial thinking, knowledge transfer
Using the property of even functions	Correlating the axis of symmetry with the axis Oy	Transfer, synthesis
Logical deduction	From the relation $1 - a = 0$ is $a = 1$.	Deductive logical reasoning
Verifying the result	Confirming that the solution satisfies the parity condition	Evaluation, argumentation

- In the first solution (5D method), the heuristic techniques of *requirement analysis*, *equivalent transformation*, *introduction of an auxiliary element (substitution)*, *generalization and verification prevail*. In other words, this is an algebraic solution.
- Variational analysis *presents heuristic techniques for studying functions, geometric interpretation, knowledge transfer, identifying an invariant, and logical deduction*. This is an analytical-geometric solution, based on the study of the variation of the function and the symmetry of the graph.

Next we solve problems and propose to school teachers and students to apply the heuristic methods explained in examples 1 and 2 to the given problems:

Example 3. For each value of the parameter t , solve the equation:

$$\sqrt{t + \sqrt{t + x}} = x.$$

Solution. The given irrational equation is equivalent to the system:

$$\begin{cases} x \geq 0; t + x \geq 0 \\ t + \sqrt{t + x} = x^2 \end{cases} \Leftrightarrow \begin{cases} x \geq 0; t \leq x^2 \\ t + x = (x^2 - t)^2 \end{cases} \Leftrightarrow \begin{cases} x \geq 0; x^2 \geq t \\ x^4 - 2tx^2 - x + t^2 + t = 0. \end{cases} \quad (1)$$

The equation in system (1) is of the fourth degree with respect to the variable x and is of the second degree with respect to the parameter t . Let us bring this equation to the form:

$$t^2 - (2x^2 + 1)t + x^4 - x = 0. \quad (2)$$

The discriminant of the square trinomial with respect to the parameter t in equation (2) depends on the variable x and is equal to:

$$D(x) = (2x^2 + 1)^2 - 4(x^4 - x) = 4x^4 + 4x^2 + 1 - 4x^4 + 4x = (2x + 1)^2$$

Therefore, the roots of equation (2) will be:

$$t_{1,2} = \frac{1}{2}((2x^2 + 1) \pm |2x + 1|) \Leftrightarrow t_1 = x^2 - x \cup t_2 = x^2 + x + 1 \quad (3)$$

Thus equation (2) can be written in the form:

$$(t - x^2 + x)(t - x^2 - x - 1) = 0 \quad (3)$$

From (3) it follows that system (1) is equivalent to the totality of two systems:

$$\begin{cases} x^2 - x - t = 0, \\ x^2 - t \geq 0, \\ x \geq 0. \end{cases} \cup \begin{cases} x^2 + x + 1 - t = 0, \\ x^2 - t \geq 0, \\ x \geq 0. \end{cases} \Leftrightarrow \begin{cases} x^2 - t = x, \\ x \geq 0. \end{cases} \cup \begin{cases} x^2 - t = -(x + 1), \\ x \leq -1, \\ x \geq 0. \end{cases} \Leftrightarrow$$

$$\begin{cases} x^2 - x - t = 0, \\ x \geq 0. \end{cases} \Leftrightarrow \begin{cases} t < -\frac{1}{4} \cup \begin{cases} -\frac{1}{4} \leq t \leq 0 \\ x_{1,2} = \frac{1 \pm \sqrt{1+4t}}{2} \end{cases} \cup \begin{cases} t > 0 \\ x_1 = \frac{1 + \sqrt{1+4t}}{2} \end{cases} \end{cases}$$

The solutions of the last system give the answer to the problem.

Answer:

- For $t < -\frac{1}{4}, x \in \emptyset$;
- for $-\frac{1}{4} < t \leq 0, x_{1,2} = \frac{1 \pm \sqrt{1+4t}}{2}$;
- for $t = -\frac{1}{4}$ and $t > 0, x_1 = \frac{1 + \sqrt{1+4t}}{2}$.

Example 4. For what values of the parameter t the equation $x - t = 2|2|x| - t^2|$ has solutions. To find the solutions of the given equation.

Solution. In the xOy coordinate plane we construct the graphs of the functions $y = x - t$ and $y = 2|2|x| - t^2|$ (Figure 2).

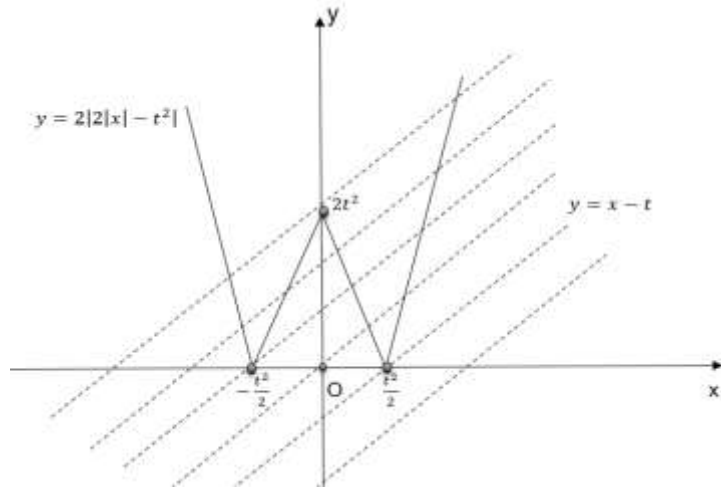


Figure 2. The graphs of the functions $y = x - t$ and $y = 2|2|x| - t^2|$

The number of solutions of the given equation will coincide with the number of common points of the graphs of the functions $y = x - t$ și $y = 2|2|x| - t^2|$. Let us observe that the family of lines $y = x - t$ intersects the axis and that of the ordinates at the corresponding points $(t; 0)$ and $(0; -t)$ and the graph of the function $y = 2|2|x| - t^2|$ has with the coordinate axes the common point $(-t^2; 0), (t^2, 0)$ and $(0; 2t^2)$ correspondingly.

From the observed we conclude: if $\frac{t^2}{2} < t$, mean $t \in (0; 2)$, then the given equation has no solutions; if $\frac{t^2}{2} = t$, mean $t = 0$ or $t = 2$, then the equation has only one solution; the equation will have two solutions if the conditions are satisfied:

$$0 < t < \frac{t^2}{2}, \left\{ -\frac{t^2}{2} < t < 0 \text{ and } \begin{cases} 2t^2 < -t \\ t < \frac{t^2}{2} \end{cases} \right.$$

Thus, the given equation will have two solutions if:

$$t \in (-\infty; -2) \cup \left(-\frac{1}{2}; 0\right) \cup (2; +\infty).$$

The given equation will have three solutions if right $y = x - t$ will pass through the coordinate point $(-\frac{t^2}{2}; 0)$ or by the point with the coordinates $(0; 2t^2)$. In case two $2t^2 = -t$, and in the first case $-\frac{t^2}{2} = t$. Thus the equation has three solutions if $t = -\frac{1}{2}$ or $t = -2$. The given equation has four solutions, if the system is compatible:

$$\begin{cases} t < -\frac{t^2}{2} \\ 2t^2 > -t \end{cases} \Rightarrow -2 < t < -\frac{1}{2}.$$

Answer: if $t \in (0; 2)$, the equation has no solutions;

if $t = 0$ or $t = 2$, the equation has a solution;

if $t \in (-\infty; -2) \cup \left(-\frac{1}{2}; 0\right) \cup (2; +\infty)$, the equation has two solutions;

if $t = -\frac{1}{2}$ or $t = -2$, the equation has three solutions;

if $t \in \left(-2; -\frac{1}{2}\right)$ the equation has four solutions.

Example 5. To find out all the values of the parameter $t \geq 1$, for each of which the area of the figure, located in the half-plane $x \geq 0$ and bordered by straight lines $y = 1$, $y = 2$ and the curves $y = tx^2$, $y = \frac{tx^2}{2}$, will it be the biggest? To find out this area.

Solution. For a fixed value of the parameter t in figure 3, the curvilinear trapezoid $ABCD$ is constructed, the maximum area of which is required to be calculated.

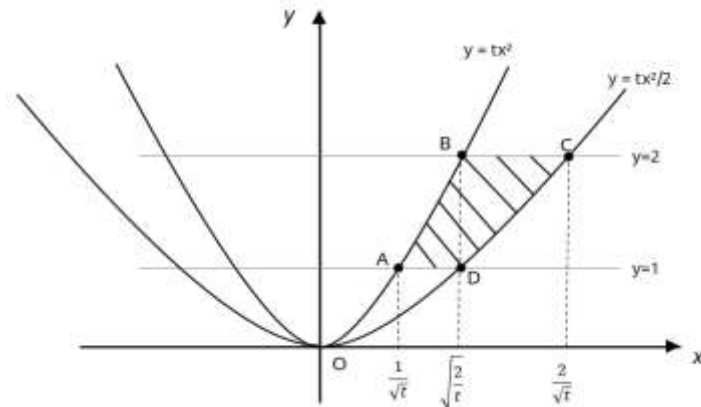


Figure 3. The construction of the curvilinear trapezoid $ABCD$

The coordinates of points A, B, C, D are determined accordingly from the solutions of the systems:

$$\begin{aligned} & \begin{cases} y = tx^2, \\ y = 1 \\ x \geq 0 \end{cases} \cup \begin{cases} y = tx^2 \\ y = 2 \\ x \geq 0 \end{cases} \cup \begin{cases} y = \frac{tx^2}{2} \\ y = 2 \\ x \geq 0 \end{cases} \cup \begin{cases} y = \frac{tx^2}{2} \\ y = 1 \\ x \geq 0 \end{cases} \\ \Leftrightarrow & \begin{cases} x_A = \frac{1}{\sqrt{t}} \\ y_A = 1 \end{cases} \cup \begin{cases} x_B = \sqrt{\frac{2}{t}} \\ y_B = 2 \end{cases} \cup \begin{cases} x_C = \frac{2}{\sqrt{t}} \\ y_C = 2 \end{cases} \cup \begin{cases} x_D = \sqrt{\frac{2}{t}} \\ y_D = 1 \end{cases} \end{aligned}$$

Since points C and D have the same abscissa, then the curvilinear trapezoid $ABCD$ consists of two parts:

1. The right triangle ABD with the curvilinear hypotenuse AB , located under the curve $y = tx^2$ and under the line $y = 2$ above the line $y = 1$, for $x \in \left[\frac{1}{\sqrt{t}}; \sqrt{\frac{2}{t}}\right]$;
2. The right triangle BCD with the curvilinear hypotenuse CD , located under the curve $y = \frac{tx^2}{2}$ and under the line $y = 2$, for $x \in \left[\sqrt{\frac{2}{t}}; \frac{2}{\sqrt{t}}\right]$.

Applying the geometric meaning of the definite integral, for the curvilinear trapezoid $ABCD$, we obtain:

$$A = \int_{\frac{1}{\sqrt{t}}}^{\sqrt{\frac{2}{t}}} (tx^2 - 1) dx + \int_{\sqrt{\frac{2}{t}}}^{\frac{2}{\sqrt{t}}} \left(2 - \frac{tx^2}{2}\right) dx = \left(\frac{tx^3}{3} - x\right) \Big|_{\frac{1}{\sqrt{t}}}^{\sqrt{\frac{2}{t}}} + \left(2x - \frac{tx^3}{6}\right) \Big|_{\sqrt{\frac{2}{t}}}^{\frac{2}{\sqrt{t}}} = \frac{1}{t} \left(\frac{10}{3} - 2\sqrt{2}\right).$$

Let us note that for $t \geq 1$, function $A(t) = \frac{1}{t} \left(\frac{10}{3} - 2\sqrt{2}\right)$ is strictly decreasing. Therefore,

$$\max_{t \geq 1} A(t) = A(1) = \frac{10}{3} - 2\sqrt{2} \text{ (s. u.)}.$$

Response: $\frac{10}{3} - 2\sqrt{2}$ (s. u.).

Conclusions

The study demonstrates that solving parameter problems requires a structured heuristic approach. They represent a didactic environment for the development of higher cognitive capacities. For ERIC-indexed didactic research, this parametric problem-solving pathway demonstrates that problems can be solved by identifying solution laws. The implementation of the 5D Heuristic Model and variational analysis cannot be understood as a simple progression of mathematical steps to solve the mathematical problem, but represents a matrix in which each algebraic action is accompanied by a specific heuristic activity aiming at the development of higher cognitive capacities. The

combined use of the 5D method and variational analysis in solving problems with parameters represents an efficient didactic and methodological strategy serving to develop higher cognitive capacities.

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