

INTERDISCIPLINARITY IN UPPER SECONDARY MATHEMATICS: A LONGITUDINAL CURRICULUM ANALYSIS ACROSS TWO GENERATIONS OF SYLLABI AND ITS IMPLICATIONS FOR INSTRUCTIONAL DESIGN

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Abstract. Why do so few upper secondary graduates recognise, outside the mathematics classroom, the structures practised for years inside it? The article argues that school mathematics suffers not from a deficit of content but from one of context and coordination. A longitudinal curriculum analysis of the Romanian syllabi for grades 9–12, across two generations of documents, identifies twelve conceptual nodes distributed along four axes. Observable indicators and three modes of implementation are added to the IMIA didactic model and illustrated through two lessons. Modelling emerges not as a compromise between rigour and relevance, but as the point where they coincide.

Keywords: mathematics education, interdisciplinarity, curriculum analysis, mathematical modelling, longitudinal curriculum, upper secondary education, critical mathematics education.

INTERDISCIPLINARITATEA ÎN MATEMATICA DE LICEU: O ANALIZĂ CURRICULARĂ LONGITUDINALĂ PE DOUĂ GENERAȚII DE PROGRAME ȘI IMPLICAȚIILE EI PENTRU PROIECTAREA DIDACTICĂ

Rezumat. De ce recunosc atât de puțini absolvenți, în afara orei de matematică, structurile exersate ani la rând în clasă? Articolul susține că matematica școlară suferă nu de un deficit de conținut, ci de unul de context și de coordonare. O analiză curriculară longitudinală a programelor pentru clasele a IX-a – a XII-a, pe două generații de documente, identifică douăsprezece noduri conceptuale distribuite pe patru axe. Modelului didactic IMIA i se adaugă indicatori observabili și trei regimuri de aplicare, ilustrate prin două lecții. Modelarea apare nu ca un compromis între rigoare și relevanță, ci ca punctul în care ele coincid.

Cuvinte-cheie: educație matematică, interdisciplinaritate, analiză curriculară, modelare matematică, curriculum longitudinal, învățământ liceal, educație matematică critică.

1. Introduction

There is a moment that almost every mathematics teacher will recognise. A student who has just solved an optimisation problem with derivatives correctly on the board asks, at the end, whether “this will be on the baccalaureate exam”. The question is not a sign of a lack of intelligence but of a lack of meaning: the student has carried out an algorithm flawlessly without ever understanding what it is for. The scene has a more instructive version, which we have met several times in the final grades. The same student, faced with the same problem phrased in the language of physics, solves it correctly in the physics lesson and declares, quite sincerely, that “in maths it was something else”. The student possesses both procedures separately; what is missing is the structure that unifies

them. Yet precisely this structure should be the outcome of learning mathematics, not a side effect of it.

The PISA 2022 assessment framework defines mathematical literacy with explicit reference to the modelling cycle [33]. What is measured internationally is thus no longer the mastery of procedures, but the capacity to mobilise them in a situation nobody has prepared in advance for mathematisation. In the same assessment, Romanian students scored 428 points in mathematics, against an OECD average of 472, and only about half reached Level 2 [34]. The description of that level does not target more difficult mathematical content but an operation: recognising, without direct instructions, how a simple situation can be represented mathematically. This is exactly the operation the student in the scene above was missing. The contrast with Romania's consistently remarkable olympiad results shows that we are not dealing with a deficit of capacity, but with a gap between what we know how to form at the top and what we manage to form at scale, in a teaching practice that remains centred on formal rigour and algorithmic drill [5; 6; 40; 31] and in a curriculum fragmented along disciplinary lines [37; 9].

We do not claim, however, that Romanian syllabi lack interdisciplinary openings; the analysis below shows precisely the opposite. The problem is that these contents are taught as successive topics, without the connection to the fields that give them meaning being made at the moment the concept is formed. Interdisciplinarity is invoked at the level of intentions and abandoned at the level of design. If the deficit were one of content, the solution would belong exclusively to curricular reform, that is, to a level of decision the classroom teacher has no access to. If, however, the deficit is one of context and coordination, a substantial part of the solution lies in instructional design, which is within the teacher's reach. The reformulation shifts responsibility to a place where it can actually be exercised.

The moment at which we write adds a further stake. After almost two decades of curricular stability, new framework plans and syllabi are being applied progressively from the 2026–2027 school year [27; 28; 29], and the mathematics syllabus makes interdisciplinary modelling the sixth general competence of the subject. This is good news, but also a test: a well-worded competence does not, by itself, produce a differently structured lesson. Between the wording of the competence and Wednesday's lesson there is a space that has to be filled with design tools, and this article is concerned precisely with that space.

We therefore ask to what extent the mathematics syllabi for grades 9–12, old and new, allow — without additional hours — a longitudinal restructuring of teaching around modelling, so that interdisciplinary connections do not remain episodic but accumulate from one year to the next. The contribution we claim is a map of the interdisciplinary nodes of the upper secondary mathematics curriculum, verified on two generations of

documents, from which four longitudinal axes follow. For the didactic dimension we draw on the IMIA model, set out at length elsewhere [1], to which we add here indicators by which its implementation can be verified rather than merely declared, and three modes of implementation compatible with the real time of a lesson.

2. Theoretical Background

The term interdisciplinarity is used in current pedagogical discourse with a generosity that blunts its meaning. The classical taxonomy distinguishes several levels of integration [18; 15; 9], and recent reviews show that in mathematics education the boundaries between them remain blurred [35]. The useful distinction for the classroom teacher, however, is not the one between labels. A mathematics lesson does not become interdisciplinary because it uses an example from physics, but because it requires the student to move effectively between the two languages and to judge whether the move is adequate. If the example can be removed without the task changing, then it is decoration.

We formulate this criterion, which we shall call the *decoration test*, as four questions the teacher can ask before declaring a lesson interdisciplinary. Is the context necessary to formulate the task, or does it merely illustrate it? Does the student have to choose what to keep and what to neglect from the situation, or are all the quantities already given? Must the result be reinterpreted in the language of the field of origin? Is there, finally, a criterion coming from the partner field, and not from mathematics, by which the solution can be declared plausible or implausible? This last is the most severe test and the one most often failed in practice. A task that passes all four filters calls for integrating ability [10]; one that passes only the first is, at best, an illustrative application. The difference is not one of degree, but of kind.

Modelling, moreover, is not an imported methodological novelty. For Hans Freudenthal, mathematics is a human activity, the mathematisation of reality, which the student must reinvent under guidance rather than receive ready-made [12]. Freudenthal distinguishes horizontal mathematisation, by which a real situation is translated into mathematical language, from vertical mathematisation, by which concepts are reorganised within mathematics. The consequence for design is counter-intuitive: a well-built interdisciplinary approach *increases* the demand for vertical mathematisation. The exponential function met in radioactive decay, bacterial growth and compound interest sooner or later calls for the property that unifies them to be stated, namely that the rate of change is proportional to the current value. This is vertical mathematisation in its purest form, and it was provoked by context, not hindered by it.

The modelling cycle of Blum and Leiß, from the situation model to validation, can be read as the didactic operationalisation of this intuition [4; 17]. It remains the dominant reference in empirical research [8], and the reason seems to us didactic rather than

theoretical: the cycle describes a path the teacher can walk with the students and observe step by step. The student who models judges, approximates and owns the limits of the model [32; 31; 42; 19].

In STEM education, mathematics is not merely a computational tool at the service of the other disciplines, but the element of epistemological coherence that unifies them [7]. Review literature nevertheless shows, with an insistence that should give us pause, that it is the least represented discipline in integrated STEM frameworks [36; 16; 14; 22]. It can be said without exaggeration that mathematics risks being the discipline most often invoked and least often integrated in the projects that carry its initial in their acronym, and the effectiveness of these projects for mathematics learning has been called into question [11; 41].

There is, however, a deeper motivation than student interest. Ole Skovsmose showed that mathematical models do not merely describe the world but actively format it: banking decisions, credit-scoring algorithms and digital infrastructures run on models that most citizens cannot examine critically [39; 2; 13]. Hence the proposal to extend the modelling cycle, after validation, with a decision that also takes ethical, economic or social aspects into account [21; 23], modelling becoming a component of citizenship education [20]. A graduate who has never practised formulating assumptions and asking what remains outside the calculation is defenceless in a world increasingly governed by algorithms.

Freudenthal thus answers the question of *where* learning starts, Blum and Leiß the question of *how* it unfolds, and Skovsmose the question of *what* it is all *for*. The model we propose sits at the point where the three answers meet; we claim originality for none of the components, but for their articulation into a design tool extended over the whole upper secondary cycle.

3. Methodology of the Curricular Analysis

The approach is analytical and interpretive. We analysed the specific competences, contents and methodological recommendations of the mathematics syllabi for grades 9–12 on two series of documents: the syllabi applied over the last two decades [30; 24; 25; 26] and the new curricular framework approved in 2025–2026 [27; 28; 29]. The second series is not a mere bibliographic update but a robustness test: nodes dependent on the wording of a particular syllabus would have been invalidated by the change of document. The nodes, however, depend not on how a competence is drafted but on the objective relation between a mathematical concept and the fields that use it. Conservation of energy does not change when the physics syllabus changes, and the derivative remains a velocity whatever ministerial order is in force.

For each year of study we retained only the contents that pass the decoration test, and we deliberately excluded associations that work only through verbal analogy, however attractive they may be didactically; a metaphor is not an interdisciplinary bridge. We then followed the cognitive continuity between years, the dimension most often missing from the lists of applications published in methodological literature and the one that explains why such lists remain ineffective: an isolated application builds nothing, because the student cannot link it to anything already understood.

Not every content becomes a node. We retained only the concepts that appear with the same formal structure in at least three distinct partner fields (see Table 1), feature explicitly in the science-track syllabus and admit an extension of meaning from one year to the next. The first condition makes the effort of contextualisation worthwhile, the second guarantees compatibility with national assessment, the third turns a list into a path. The small number of concepts that results is not a disappointing outcome but the very condition of applicability: a model that declared thirty contents interdisciplinary would no longer be a model, but a list of good intentions.

The analysis was carried out by a single researcher, and another analyst might include or exclude borderline concepts such as trigonometry; a syllabus is a prescriptive text, not a description of practice; and, above all, curricular analysis establishes a possibility, not an effect.

4. Results

4.1. *Interdisciplinary nodes from grade 9 to grade 12*

In grade 9 the interdisciplinary potential is more discreet, but not absent. Elements of logic directly support reasoning in computer science, and this bridge is, of all those identified, the least exploited, although its cost is almost zero: De Morgan's laws taught in mathematics and the compound conditions of a decision statement taught in computer science are the same object, and simply stating this identity in both lessons produces an effect out of all proportion to the effort. The linear function describes uniform rectilinear motion and linear costs, but what is missing in ordinary practice is not the possibility of the connection, but the moment at which it is made. The linear function is taught in the second semester, uniform rectilinear motion in physics in the first, and the student has no reason to link them. A fifteen-minute conversation at the start of the year, in which the two teachers compare the order of their topics and move one by two or three weeks, is enough. Geometric progressions deserve a separate mention, because they prepare the most important node of the cycle: a student who has understood that a geometric progression describes growth by a constant factor already holds, in discrete form, the idea of exponential growth.

Grade 10 marks a qualitative leap. The exponential function is, in our view, the richest conceptual node of the whole upper secondary cycle: radioactive decay, the growth of a population of micro-organisms, compound interest, the spread of information or of an epidemic. One mathematical concept, four partner fields, the same formal structure. Students can question public growth models with their own tools [38]. The logarithmic function finds its meaning in scientific scales, from pH in chemistry to seismic magnitude and decibels, and here classroom experience constantly confirms one thing: students manipulate the properties of logarithms correctly long before they understand why humanity needed them. A lesson that opens by asking how the intensity of a rustle and that of a jet engine, quantities differing by twelve orders of magnitude, can be shown on the same graph produces the logarithm as a necessity, not as a definition. Probability and descriptive statistics are, in turn, the curricular place where critical mathematics education can be practised without any addition: an election poll discussed in class, with its margin of error and sampling assumptions, is at once syllabus content and an exercise in citizenship. Early on, starting situations must be observable. A grade 9 student can estimate how much their school bag weighs and check it; they cannot do the same with the capital flows of an economy.

In grade 11, matrices and linear systems discreetly introduce the idea of a system, a whole in which changing one element affects everything else, but this node remains weaker than the others. The derivative is the second major node of the cycle, being at once velocity in physics, growth rate in biology, marginal cost in economics and the slope of a curve in geometry. The difficulty students face, however, is not computational, but lies in identifying *what* is being differentiated with respect to *what*. A student who correctly writes the derivative of a composite function may be stuck when asked how fast the area of a circular oil slick is growing, not because they do not know the chain rule, but because they have not built the reflex of asking which quantities are related. This reflex is formed only through the practice of modelling, not through repeated calculation.

Grade 12 brings algebraic structures, which at first sight seem the least interdisciplinary content of the syllabus. The appearance is misleading: modular arithmetic and the group of units underpin public-key cryptography, and therefore the security of every digital transaction. In a military high school, where communications security has direct professional significance, this node has a motivational force few other contents can match. The definite integral, the last major node, allows a synthesis at the end of upper secondary school that few students reach spontaneously: differentiation and integration are two faces of the same relation between a quantity and its rate of change. It deserves a dedicated lesson, which no syllabus provides and which needs only the reorganisation of a revision lesson. The differential equations in the new syllabus finally close, elegantly, the circle opened in grade 10: radioactive decay, studied there as a given

exponential function, now appears as the solution of an equation that expresses the physical hypothesis itself. The student thus understands that the exponential formula was not a convention, but the consequence of a hypothesis they can now write down themselves.

Table 1 summarises the analysis.

**Table 1. Interdisciplinary conceptual nodes
of the upper secondary mathematics curriculum, science track**

Conceptual node	Grade	Partner fields	Extension of meaning	Axis
Linear function	9	Physics, economics, chemistry	Becomes a special case of the differentiable function (11)	Functional-dynamic
Geometric progression	9	Economics, biology, physics	Precedes the exponential function (10)	Functional-dynamic
Propositional logic	9	Computer science, philosophy, law	Precedes algebraic structures (12)	Logical-structural
Exponential function	10	Physics, chemistry, biology, economics, epidemiology	Becomes the solution of a differential equation (12)	Functional-dynamic
Logarithmic function	10	Chemistry, geophysics, acoustics, computer science	Remains a scaling tool in all later analyses	Functional-dynamic
Probability	10	Demography, epidemiology, social sciences, genetics	Extends to conditional probability and decision (12)	Probabilistic
Descriptive statistics	10	Social sciences, biology, economics	Basis for the critical interpretation of public data	Probabilistic
Linear systems	11	Economics, chemistry, engineering, computer science	Revisited through geometric interpretation in space (12)	Logical-structural
Derivative	11	Physics, biology, economics, geometry	Linked to the integral by the fundamental theorem (12)	Functional-dynamic / Optimisation
Optimisation with derivatives	11	Economics, engineering, applied geometry	Precedes decision models (12)	Optimisation and decision
Algebraic structures	12	Computer science, cryptography, digital security	Closes the axis opened by logic in grade 9	Logical-structural
Definite integral	12	Physics, chemistry, biology, economics	Synthesises the whole functional-dynamic axis	Functional-dynamic

Source: author's analysis of the curricular documents listed in Section 3

4.2. What the new curriculum brings

The new mathematics syllabus for the science track organises the subject around six general competences, the last being *Mathematical modelling of a given situation by integrating knowledge from different fields* [28]. It does not appear as a methodological suggestion but as a structural element, broken down into specific competences all the way to radioactive decay and thermal evolution treated as differential equations in grade 12; even the use of an AI assistant is allowed only on condition that the student critically

validates the solution and can explain its limits. All the nodes in Table 1 are found in the new document, and the thesis of this article thus receives institutional confirmation. The stakes shift, however: the question is no longer whether the syllabus allows it, but through which design tool a general competence becomes a lesson sequence. For military colleges, the military mathematics–computer science specialisation, with its own specialised curriculum over all four years, opens a rare coherence: navigation, ballistics, meteorology, cryptography and logistics are not motivational devices here, but professional content.

4.3. The four longitudinal axes

The nodes in Table 1 fall, without this distribution having been imposed, on four axes that cross the whole upper secondary cycle, with the derivative at the intersection of two of them (Figure 1). They emerged from the analysis rather than being fixed in advance.

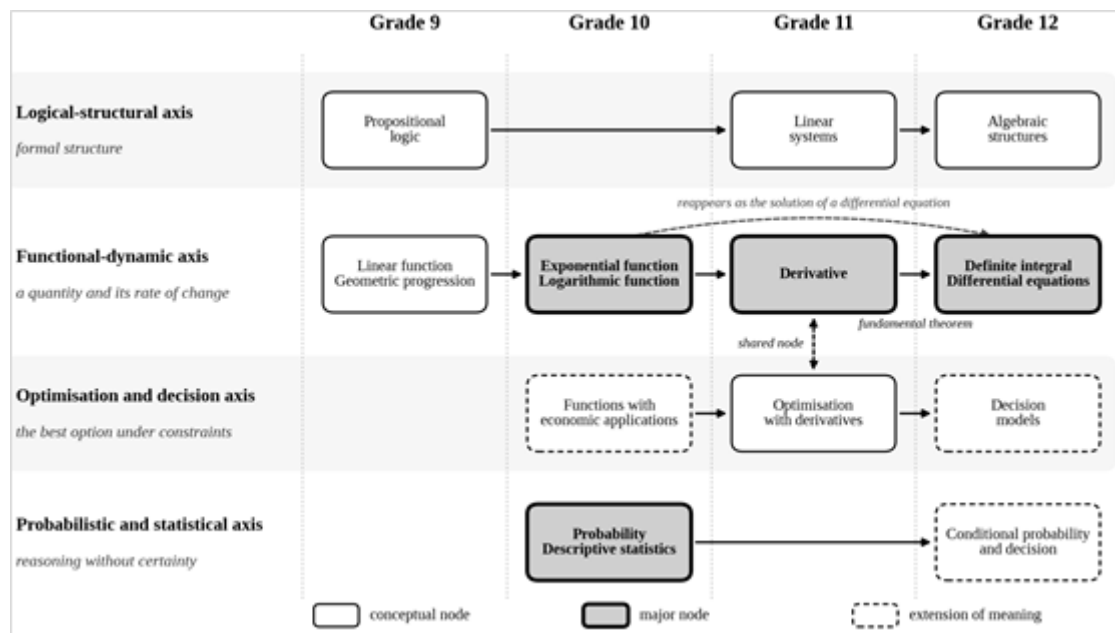


Figure 1. The four longitudinal axes of the upper secondary mathematics curriculum and the conceptual nodes that compose them

Source: developed by the author

The logical-structural axis leads from propositional logic in grade 9, through linear systems, to abstract algebraic structures in grade 12, and its unifying idea is formal structure: a set of objects with operations obeying precise rules, whatever the nature of the objects. The functional-dynamic axis runs from the first functions to integrals and differential equations, and its idea is the relation between a quantity and its rate of change; it is the richest axis and, not by chance, the one containing three of the four major nodes. The probabilistic and statistical axis starts from probability in grade 10 and reaches decision under uncertainty, and its idea, reasoning in the absence of certainty, has the most direct civic value of all. The optimisation and decision axis links functions with

economic applications to optimisation with derivatives and to decision models, around choosing the best option under given constraints.

The axes do not replace the topic structure of the syllabus; they cross it. A teacher who teaches the derivative knows that they are teaching at the same time the third moment of the functional-dynamic axis and the first serious moment of the optimisation axis, and this awareness changes the emphases of the lesson without changing its content. When a node is introduced, the teacher makes explicit its link with its previous occurrence and announces the next one, a two-minute investment that changes the cognitive status of the content. And any element that cannot be applied by a single teacher, with a textbook and a board, in a fifty-minute lesson must be either reformulated or dropped; this is a constraint we impose on ourselves.

4.4. From map to lesson: the IMIA model

The axes describe the path of the contents, but say nothing about how the lesson is built. For this we draw on the IMIA model [1], whose four cores are not successive stages but mutually conditioning dimensions: interdisciplinarity opens, modelling transforms, integration links the lesson to the past and future of the same concept, and applicability treats the mathematical result as a basis for decision rather than as an end point. The resulting sequence has eight moments (problem situation, exploration, selection of variables, mathematical model, solution, interpretation, decision, critical reflection) and fully contains the Blum–Leiß cycle, without stopping at validation. The extended cycle proposed by Maass and her colleagues goes in the same direction [21; 23], but it describes a *task*, built around controversial socio-scientific problems; we link it to a four-year *pathway* and to the ordinary mathematics lesson, where the decision may just as well be technical or economic.

The curricular analysis adds to the model something it did not contain on its own. If nodes are few and identifiable, the implementation of the model can be verified rather than merely declared. Table 2 proposes indicators formulated as observable facts, each accompanied by a counter-indicator, the concrete sign that the core is missing even though the lesson appears to follow the model.

Table 2. Observable indicators of the IMIA cores

Core	Indicator present	Counter-indicator (sign that the core is missing)
Interdisciplinarity	The starting situation is stated before the mathematical concept is named	The lesson title is written on the board at the start
Interdisciplinarity	Removing the context would make the task impossible to formulate	The context appears only in the statement and disappears from the solution
Modelling	Simplifying assumptions are written down explicitly by students	All necessary quantities are given in the statement
Modelling	At least two alternative models are compared	The first model proposed is accepted

Core	Indicator present	Counter-indicator (sign that the core is missing)
Integration	Explicit reference is made to a previous occurrence of the node	The concept is treated as new although met before
Integration	The student moves between at least two representations of the concept	Work is done exclusively in the algebraic register
Applicability	The student makes a choice between at least two options	The lesson ends with a numerical result
Applicability	Conditions under which the conclusion would no longer hold are stated	The result is presented as definitive

Source: developed by the author

The counter-indicators are shortcuts that time pressure makes reasonable. Writing the lesson title on the board at the start is not bad practice in general; in a modelling lesson, however, the student learns which tool will be used before meeting the problem that requires it.

Modelling competences develop through the combination of holistic and atomistic tasks [3], so we propose three modes. The full mode, in eight moments, is reserved for the major nodes, in four to six lessons a year. The partial mode costs ten to fifteen minutes: at the end of an ordinary lesson of exercises on derivatives, the teacher gives a numerical result obtained earlier and asks students only to say what it means in its context and whether it is plausible. The signalling mode costs two minutes: “this function is the same as the one in radioactive decay in grade 10; in two years you will meet it again as the solution of a differential equation”. Nothing is modelled, but the integration core is fed. Its minimal tool is the *node map*, a page in the student’s portfolio on which each node appears with the contexts in which it was met, the common formula and the date of each encounter. Its absence explains, we believe, why so many well-intentioned interdisciplinary projects leave no trace: a successful project, run once a semester, remains a memorable experience and an isolated event [35].

4.5. Two lessons

The first lesson, in grade 11, is run in the full mode. Students receive a single request: a company wants a 330 ml cylindrical can that uses as little sheet metal as possible. They are given no formulas and no indication of what should be differentiated. The teams first examine a real can and estimate, before any calculation, whether the optimal shape would be taller and narrower or shorter and wider than the one in their hands; experience shows that most anticipate the direction correctly, but none anticipates the ratio. Then come the assumptions: the sheet has constant, negligible thickness, the can is a right circular cylinder, and cutting waste from stamping is ignored. This last assumption, rarely stated spontaneously, is worth prompting by the teacher, because it will return at the end. From the constraint $V = \pi r^2 h = 330 \text{ cm}^3$ and the total surface area we obtain the function:

$$A(r) = 2\pi r^2 + \frac{2V}{r}, \quad r > 0,$$

and this is usually where it becomes clear whether the notion of dependent variable has really been understood. Setting the derivative equal to zero, $A'(r) = 4\pi r - 2V/r^2 = 0$, gives

$$r^* = \sqrt[3]{\frac{V}{2\pi}} \approx 3.745 \text{ cm}, \quad h^* = \frac{V}{\pi r^{*2}} \approx 7.490 \text{ cm},$$

and $A''(r) = 4\pi + 4V/r^3 > 0$ confirms the minimum. The most striking result, however, is not the numerical value but the ratio: $h^* = 2r^*$, the optimal height equals the diameter, so the axial cross-section of the minimum-area cylinder is exactly a square. As a numerical example, a cylindrical can with a volume of 330 cm³ and a radius of about 3.3 cm has a height of about 9.65 cm and a surface area of about 268.42 cm², some 1.5% above the minimum of 264.36 cm². The decision question follows naturally: would you advise the manufacturers to switch to the optimal shape? A 1.5% saving in raw material, multiplied by billions of units, is a considerable sum; on the other hand, other dimensions may be preferred for reasons of mechanical strength, transport, ergonomics and production-line design. Students have to choose and argue. The final reflection brings back what was left outside the model, above all the stamping waste neglected at the start, and reveals a rarely discussed property: near the minimum the function A is very “flat”, so that noticeable deviations from the optimal radius produce negligible increases in material. In applications, a range of near-optimal dimensions can therefore be accepted once a tolerance for the increase in area is set; the mathematical minimum remains unique.

The second lesson, in grade 10, lasts about thirty minutes. A bacterial culture starts from 1000 cells and the population doubles every 20 minutes; how many cells will there be after 24 hours? Students write $N(t) = N_0 \cdot 2^{t/T}$ without difficulty, find 72 doublings and obtain $N \approx 4.7 \cdot 10^{24}$ cells. The teacher then asks them to convert the result into mass, knowing that a bacterium weighs about one picogram:

$$m \approx 4.7 \cdot 10^{24} \cdot 10^{-12} \text{ g} = 4.7 \cdot 10^{12} \text{ g} \approx 4.7 \text{ million tonnes.}$$

In a test tube. The moment the class realises what it has calculated is, in our experience, one of the most productive of the whole year, because nobody has made a mistake: the calculation is flawless and the result is absurd. The absurdity does not come from mathematics but from the silent assumption that resources are unlimited. In response, the logistic model is introduced,

$$N(t) = \frac{K}{1 + \left(\frac{K}{N_0} - 1\right) e^{-rt}}, \quad K = 10^9, \quad r = \frac{\ln 2}{T},$$

which practically coincides with the exponential model for the first four hours but is exceeded by it by about 26.2% at six hours (relative to the logistic value) and by a factor

of almost eighteen at eight hours. The students formulate the conclusion themselves: a model is not right or wrong in itself, but adequate or inadequate on a certain domain, and the task of whoever uses it is to know where the boundary lies. In grade 12 the same structure will reappear as the equation $N'(t) = rN(t)$, and the student will understand that the exponential function was not chosen but derived; signalling this costs thirty seconds and turns an isolated lesson into a moment of a pathway.

5. Discussion

The most visible benefit of this approach is the increased perceived relevance of mathematics, but it is not the most important one. More substantial, in our view, is the strengthening of transfer: a student who built an exponential model for radioactive decay and met it again in the growth of a bacterial population recognises the structure, not just the formula, and this recognition is exactly what was missing in the scene that opened the article. The effect appears only over time, hence the longitudinal design. The operations modelling demands are not “easier” than calculation but harder [32; 8]: a student who is good at calculation may fail completely at formulating an assumption, and this failure is informative rather than discouraging, because it shows exactly where their education has been incomplete.

The limitations are real, and we state them plainly. Four to six full lessons mean eight to twelve hours a year, taken from repetitive practice, and this transfer is a deliberate decision, not a free saving [11]. Coordination with partner disciplines runs up less against a lack of training [35] than against the absence of a framework in which the two teachers can meet: one joint departmental hour per semester would achieve more than a twenty-hour training course. The most serious limitation is assessment: as long as the exam favours algorithmic items, every hour invested in modelling remains in tension with it, and this tension cannot be resolved at classroom level. The risk of false contexts, in which the real problem is merely the wrapping of an ordinary exercise, is reduced but not removed by the decoration test and Table 2. Finally, the model stems from the experience of a single teacher in a single type of school, and its generalisation cannot be argued, only verified through replication.

The most serious objection comes from mathematicians: excessive contextualisation would dilute rigour, turning mathematics into a collection of applied recipes. It has a real basis, and the literature warns that STEM integration can reduce mathematics to an auxiliary tool [36]. We nevertheless maintain that, at the level of modelling, the opposition between rigour and relevance is false. Rigour, in the strict mathematical sense, means that every statement is justified by explicit premises. In a lesson of algorithmic drill, the premises are given by the statement and remain unexamined, and the student is rigorous only within a framework they did not build and do not control. In a modelling

lesson, the premises must be formulated by the student, and the demand for justification extends to them. The bacteria scenario shows exactly this: the calculation is flawless and the conclusion is false, because a premise was not stated, and no computational check would have detected the error. What we call “rigour” at school is often, in fact, *accuracy*, the absence of computational errors in a chain of operations. Accuracy is necessary, but it is not rigour; rigour presupposes awareness of premises, and this is formed through modelling or not at all. A curriculum organised solely around exercises is not more rigorous, only more accurate, which is something else entirely.

The benefits above remain working hypotheses, theoretically grounded, not empirically confirmed results. We have shown that the syllabi, in both generations, contain the identified nodes and that these are distributed along four axes; showing that teaching organised around them produces better results requires a longitudinal study of a cohort over four years, with transfer items and with Table 2 used as an observation grid.

6. Conclusions

The mathematics syllabi for grades 9–12 contain, in every year of study, contents with explicit interdisciplinary potential that teaching practice exploits only partially. The deficit is therefore not one of content but of context and coordination, and this reformulation moves the solution from curricular reform, beyond the teacher’s reach, to instructional design, within it. The selection criterion led to twelve conceptual nodes, distributed along four longitudinal axes and found in both generations of syllabi. The new curriculum confirms the thesis through its sixth general competence, but shifts the stakes towards the design tool, and the IMIA model, completed with observable indicators and three modes of implementation, offers one form of that tool.

For military education, in which we work, this carries additional weight. A military college student does not need to be convinced that trigonometry serves indirect measurement or that modular arithmetic underlies communications security. This privileged situation also brings a responsibility: military colleges can serve as a validation ground for interdisciplinary models which, once verified, can be transferred to the regular science track.

The conclusion we draw is deliberately modest. Such an approach does not by itself solve the crisis of meaning in school mathematics, whose causes lie far beyond the classroom, but it offers the classroom teacher a concrete, replicable, syllabus-compatible tool to mitigate it. We return, finally, to the opening scene. The student who asks whether optimisation with derivatives “will be on the baccalaureate” is not really asking about the exam. They are asking whether what they are doing has a purpose. What we propose is not an answer to that question, but a restructuring of the lesson in which the question no longer arises.

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