

## ON THE ABILITY OF ENGINEERING STUDENTS TO GET CORRECT ANSWERS IN MATHEMATICS EXAMINATIONS

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**Abstract.** The results of engineering students in mathematics examinations where marks are awarded only for correct answers are compared with those where partial marks are awarded. Data from two courses are analysed and conclusions are drawn.

**Keywords:** traditional examinations; partial marks; correlation coefficient; linear regression.

## DESPRE CAPACITATEA STUDENŢILOR LA INGINERIE DE A OBTINE RĂSPUNSURI CORECTE LA EXAMENELE DE MATEMATICĂ

**Rezumat.** Rezultatele obținute de studenții ingineri la examenele de matematică la care se acordă note doar pentru răspunsuri corecte sunt comparate cu cele la care se acordă note parțiale. Sunt analizate date de la două cursuri și se trag concluzii.

**Cuvinte-cheie:** examene tradiționale, note parțiale, coeficient de corelație, regresie liniară.

### 1. Introduction

At the start of the COVID-19 pandemic, teachers had to adapt quickly to the new situation. In particular, examinations were sometimes given online, often without any supervision.

At Polytechnique Montréal, several professors opted for multiple choice examinations in the winter term of 2020. The author, however, chose to give traditional examinations at home. But, instead of asking students to scan their solutions and upload them to the course Moodle site, he asked them to send only their answers to the questions.

The main reason for this decision was that the authorities asked professors to give students 45 minutes at the end of the examinations to scan and upload their solutions. However, depending on their computer equipment and skills, the actual time required to perform these operations could vary greatly from one person to another, which could therefore create unfairness. Writing only the answers to the questions would not only take a much shorter time, but also a much more uniform time for the students.

This way of evaluating students is different from multiple-choice examinations, since students have to propose an answer themselves. Thus, luck (or bad luck) does not come into play. The effectiveness of multiple-choice examinations has been the subject of several research papers; see, for example, [4]. The author has used this type of test for several years in a course on probability, which is a particularly suitable topic.

When marking the examinations, except for insignificant errors, students were only awarded marks for correct answers. Since partial marks were not awarded, as when

students hand in their complete solutions, it was obviously to be expected that the results would be lower than usual.

At Polytechnique Montréal, a letter system is used for final grades: F, D, D<sup>+</sup>, C, C<sup>+</sup>, B, B<sup>+</sup>, A and A\*. The thresholds required to obtain these grades are not fixed in advance. The professor responsible for the course sets the threshold for a D and that for an A. The other thresholds, except the one for an A\*, are then set automatically. For example, if an average of 10 out of 20 is required for a D and 16 for an A, then the thresholds for the other letter grades are 11, 12, 13, 14 and 15. The threshold for the A\* is set independently. Students' average is out of 4. A letter grade D is worth 1 point, a D<sup>+</sup> 1.5 points, ..., and an A gives 4 points, as does an A\*.

Because the professor can choose the thresholds taking into account the difficulty of the examinations and other factors that may have influenced the results, having lower grades in the final examination should not penalise the students, as only the letter grade obtained in the course appears in their report cards.

Polytechnique Montréal is an engineering school affiliated with the Université de Montréal, which is placed around 100th in various rankings of the world's best universities. As a result, the level of the courses is high. In addition, students have generally obtained very good results at the pre-university level, especially in mathematics.

In Section 2, the results obtained by the students in the two courses the author taught in the winter term of 2020 will be presented and the data analysed. Some concluding remarks will be made in Section 3.

## **2. Results**

In the winter term of 2020, the author taught two courses, namely MTH1110: Ordinary Differential Equations and MTH2303: Probability and Statistics. These are both undergraduate courses.

MTH1110 is taken by Civil Engineering students only. These students have an average level of mathematics compared to the Polytechnique student body. Previously, Chemical Engineering students took MTH1110. Their level is also average in mathematics. MTH2303 is taken by Electrical Engineering and Engineering Physics students, who are above average in mathematics.

The author has been teaching the MTH1110 course every winter for several years. Therefore, he can easily compare the results of the winter term of 2020 with those of previous terms. On the other hand, he had not taught MTH2303 for several years, in spite of the fact that his area of specialization is applied probability; he taught another version of the course. As a result, comparison with previous terms is more difficult.

For each course, the author used as a reference book a textbook that he has written specifically for that course ([2] and [3]). In these manuals, there are many exercises taken

from previous examinations. Thus, it is possible to check the level of the courses by consulting these manuals.

In the winter term of 2020, the author composed slightly different final examinations, to take into account the evaluation system used. Instead of having sometimes quite long questions worth 4 or 5 marks out of 20, the examinations had several short questions or sub-questions, usually worth 1 mark out of 20. However, the level was the same as usual.

At Polytechnique, it is traditional to mark examinations out of 20. In addition, the marks for each question or sub-question are usually rounded to the nearest half mark. Thus, for a question worth 1 mark, there are only three possible marks: 0, 0.5 or 1. In the winter term, the only possibilities were 0 or 1 (out of 1).

**Table 1. Scores (out of 20) obtained by MTH1110 students in the mid-term and final examinations, as well as the correlation coefficient  $r$  of the scores**

Year	Mid-term	Final	$r$
2015	10.139	9.600	0.617
2017	9.803	8.351	0.598
2018	11.173	11.634	0.645
2019	10.481	8.354	0.812
2020	10.438	7.810	0.533

**Table 2. Average scores for four subgroups in MTH1110, as well as the average percentage decrease on the final examination compared to the mid-term**

Subgroup	Mid-term	Final	% of decrease
I	5.875	4.750	19.15
II	11.222	8.060	28.18
III	13.583	9.420	30.65
IV	15.917	9.917	37.70

The final examination in both courses took place at home, without supervision. In order to minimise the risk of plagiarism, each student was given a different, but equally difficult, version of the final examination. This was possible because the size of two groups was relatively small: 29 students took the final examination in MTH1110, and 16 in MTH2303. The standard at Polytechnique for many core courses is 60 students.

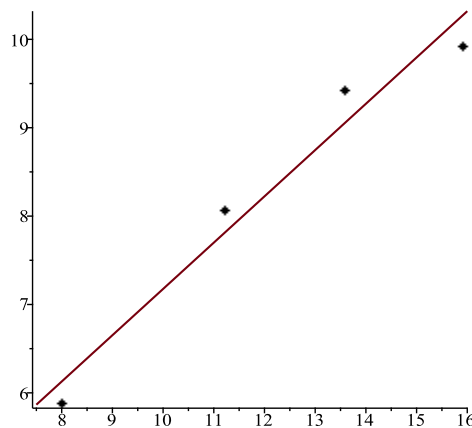
In the MTH1110 course, there is only a mid-term examination worth 45% of the final grade, and the final examination worth 55%. In Table 1, the results obtained by students in

these two examinations in 2020 and in previous years are presented. In addition, the correlation coefficient  $r$  of the scores obtained by the students who took both examinations is given. It can be seen that the average of the 2020 cohort in the mid-term examination is very slightly higher than the average of the other cohorts (10.4). So, this was a typical cohort. On the other hand, its average in the final examination is 1.675 marks lower than that of the other cohorts, which is not surprising. In addition, the correlation coefficient is about 20% smaller than the average for the other years, which was less expected.

Now, to see the effect of not giving partial marks on students of various levels, the group was divided into four subgroups, according to the results obtained in the midterm examination: I (Weak):  $[0,8.0]$ , II (Average):  $(8.0,12.5]$ , III (Good):  $(12.5,15.0]$  and IV (Very good):  $(15.0,20]$ . These subgroups have 8, 9, 6 and 6 students, respectively. The average scores for each of these subgroups are presented in Table 2 along with the average percentage decrease on the final examination compared to the midterm. In addition, Figure 1 shows the regression line corresponding to these data.

From the data in Table 2 and the regression line, we can draw the following conclusions:

- Students who scored low in the mid-term examination were still able to correctly solve some (probably the easiest) questions in the final examination. So, the few marks they got in the mid-term examination were not necessarily just partial marks awarded for their efforts.



**Figure 1. Regression line corresponding to the data in Table 2**

- Students who had done very well in the mid-term examination, and who were heading for an A or even an A\* in the course, were very much affected by not receiving partial marks, contrary to what might have been expected. These students, on average, were only able to solve less than half of the final examination questions correctly, which is really disappointing. The majority of them probably still got at least an A, but this A is more due to the deviation of their grades from the general average than to the excellence of their results. In their defence, examinations at Polytechnique Montréal often require students to work very quickly. Moreover, it is

generally better for them to give a solution, even a partial one, to each question than to have correct answers to some questions and no answers at all to others. This is also the case in many engineering schools, particularly in the French education system.

- Although the correlation coefficient between the students' scores in the two examinations is not very high, the regression line shows that the scores in the midterm examination are a very good predictor of the category in which a given student will be in the final examination. Indeed, the coefficient of determination (see, for instance, [1]),  $R^2$ , is equal to 96.1%.

In MTH2303, there were two mid-term examinations. The combined average in these two examinations was 9.720 out of 20, and the average in the final exam was 8.156, a drop of 16.09 percent (compared to 25.18 in MTH1110). In addition, the correlation coefficient of the scores was 0.579. It is worth noting that the correlation coefficient of the scores in the two mid-term exams was only 0.304. Thus, the way in which students are assessed does not have a great influence on this correlation coefficient.

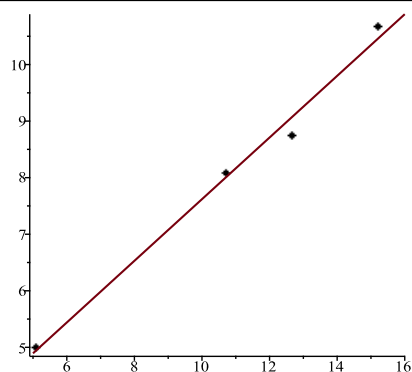
Table 3 presents the combined average scores in the two mid-term examinations for the four subgroups defined as above, the average in the final examination and the percentage decrease in scores; the regression line is shown in Figure 2. The four subgroups have respectively 3, 6, 4 and 3 students. Although these numbers are small, we can see that the results are consistent with those obtained in MTH1110. This time, the coefficient of determination is equal to 99.1%.

### 3. Concluding remarks

In this note, we presented the results obtained by engineering students in the final examination in two mathematics courses when there were no partial marks awarded for their solutions. As we have seen, even the best students did not manage to solve more than about half the questions correctly in this final examination.

**Table 3. Average scores for four subgroups in MTH2303, as well as the average percentage decrease on the final examination compared to the mid-terms**

Subgroup	Mid-terms	Final	% of decrease
I	5.100	5.000	1.96
II	10.717	8.080	24.60
III	12.675	8.750	30.97
IV	15.217	10.670	29.88



**Figure 2. Regression line corresponding to the data in Table 3**

There are various reasons for this, not least the fact that students are asked to work quickly during examinations, and also the habit they have of trying to answer as many questions as possible rather than trying to solve as many questions correctly as possible.

It would normally be difficult to get students to accept this way of assessing them. Many take it for granted that as soon as they have written something, they should receive at least partial marks for their work. Because of the evaluation system used at Polytechnique Montréal, the author was able to attempt this experiment without penalising students with respect to their final grades.

One advantage of not awarding partial marks in examinations would be to encourage students to strive for excellence. Many seem to be satisfied with having *the idea of the solution* or having it *almost right*.

One wonders whether this tradition of making students work quickly, rather than giving them sufficient time to think, as is more the norm in pure mathematics departments courses, should be continued. It is difficult for one professor to try to change the system. One can only hope that when the students become engineers, time will be less of a constraint and they will be able to complete their work without making mistakes that can have serious consequences.

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