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On a method of constructing topological quasigroups obeying certain laws

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Abstract. A new method of constructing non-associative topological quasigroups obeying certain laws is given. Also, in this paper we research T-quasigroups with Abel-Grassmann identity $(ab) \cdot c = (cb) \cdot a$.

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Keywords: T-quasigroup, AG-quasigroup, GA-quasigroup, Manin quasigroup, Cote quasigroup, medial, semimedial, paramedial and bicommutative quasigroup, topological quasigroup.

Despre o metodă de construcţie a quasigrupurilor topologice care îndeplinesc anumite identităţi

Rezumat. În lucrarea dată este prezentată o nouă metodă de construcţie a quasigrupurilor topologice neasociative care respectă anumite legi. Totodată sunt cercetate T-quasigrupurile care satisfac identitatea Abel-Grassmann $(ab) \cdot c = (cb) \cdot a$.

Cuvinte-cheie: T-quasigrup, AG-quasigrup, GA-quasigrup, quasigrupul Manin, quasigrupul Cote, quasigrup medial, semimedial, paramedial şi bicomutativ, quasigrup topologic.

1. Introduction

In this paper, two central issues were examined.

Problem 1. Let $(Q, \cdot)$ be a $T-quasigroup$. Under which conditions the Q is a quasigroup (of its T - forms $(Q(+), \varphi, \psi, a))$ satisfying the identities P_i of some algebraic structure, where $i = 1, 2, ..., k$?

In the condition of problem formulated above we research $T - quasigroups$ with Abel-Grassmann identity $(ab) \cdot c = (cb) \cdot a$.

It is shown that if G is a $T - quasigroup$, then G is $AG - quasigroup$ if and only if for any of its $T - forms$ $(G(+), \varphi, \psi)$ is $\varphi^2(x) = \psi(x)$.

At the same time in this paper we examine the following problem.

Problem 2. Let $(G, +)$ be a commutative topological group. Under which conditions on the set $G \times G$ can be defined the binary operation $(\circ)$ such that $(G \times G, \circ)$ is a non-associative topological quasigroups obeying certain laws?

Our main goal is to prove a new method of constructing topological quasigroups.

The authors have used the concept of a special direct product of a topological Abelian group G and proved that a binary operation can be defined on the set $G \times G$, such that the new algebraic structure is a non-associative topological quasigroups obeying certain laws.

Thus, solving the problem formulated above, it was demonstrated that any commutative topological group can be "transformed" into non-associative topological quasigroup obeying certain laws using the method developed. Examples of quasigroups that satisfy the examined identities were constructed.

The results established are related to the results of L. Chiriac and N. Josu in [1, 2] and to the research papers [3, 4, 8, 9, 10].

2. Basic Notions

In this section we recall some fundamental definitions and notations [5, 6, 7, 11].

A non-empty set G is said to be a *groupoid* with respect to a binary operation denoted by $\{\cdot\}$, if for every ordered pair (a, b) of elements of G there is a unique element $ab \in G$.

A quasigroup is a binary algebraic structure in which one-sided multiplication is a bijection in that all equations of the form $ax = b$ and $ya = b$ have unique solutions.

A groupoid G is called a primitive groupoid with divisions, if there exist two binary operation $l : G \times G \to G$, $r : G \times G \to G$ such that $l(a, b) \cdot a = b, a \cdot r(a, b) = b$ for all $a, b \in G$. Thus, a primitive groupoid with divisions is a universal algebra with three binary operations.

A primitive groupoid G with divisions is called a quasigroup if the equations $ax = b$ and $ya = b$ have unique solutions. In a quasigroup G the divisions l, r are unique. If the multiplication operation in a quasigroup $(G, \cdot)$ with a topology is continuous, then G is called a semitopoligical quasigroup. If in a semitopological quasigroup G the divisions l and r are continuous, then G is called a topological quasigroup.

An element $e \in G$ is called an *identity* if $ex = xe = x$ every $x \in G$. A quasigroup $(G,)$ with an identity element $e \in G$ is called a loop.

A groupoid $(G, \cdot)$ is called *medial* if it satisfies the law $xy \cdot zt = xz \cdot yt$ for all $x, y, z, t \in G$. A groupoid $(G, \cdot)$ is called *paramedial* if it satisfies the law $xy \cdot zt = ty \cdot zx$ for all $x, y, z, t \in G$.

A groupoid $(G, \cdot)$ is called *bicommutative* if it satisfies the law $xy \cdot zt = tz \cdot yx$ for all $x, y, z, t \in G$.

A groupoid $(G, \cdot)$ is called *AD-groupoid* if it satisfies the law $a \cdot bc = c \cdot ba$ for all $a, b, c \in G$.

A groupoid $(G, \cdot)$ is called a *groupoid Abel-Grassmann* or *AG-groupoid* if it satisfies the left invertive law $(ab) \cdot c = (cb) \cdot a$ for all $a, b, c \in G$.

A groupoid $(G, \cdot)$ is called a *GA-groupoid* if it satisfies law $(ab) \cdot c = c \cdot ba$ for all $a, b, c \in G$.

A groupoid $(G, \cdot)$ is called a *groupoid Manin* or *CH-groupoid* if it satisfies the law $x(y \cdot xz) = (xx \cdot y)z$ for all $x, y, z \in G$.

A groupoid $(G, \cdot)$ is called a *groupoid Cote* if it satisfies the law $x(xy \cdot z) = (z \cdot xx)y$ for all $x, y, z \in G$.

Left semi-medial identity in a groupoid $(G, \cdot)$ has the following form: $xx \cdot zt = xy \cdot xz$ for all $x, y, z, t \in G$. R.H. Bruck [14] used this identity to define commutative Moufang loops in the class of loops.

3. T-quasigroups with Abel-Grassmann identity

In this section we study some aspects of charaterization of abelian groups isotopic to $T - quasigroups$.

Definition. Quasigroup $(G, \cdot)$ is a $T - quasigroup$ if and only if there exists an abelian group $(G, +)$, its automorphisms φ and ψ, and a fixed element $a \in G$ such that $x \cdot y = \varphi(x) + \psi(y) + a$ for all $x, y \in G$.

Under the conditions of Definition we shall say that the isotope $(G, \cdot)$ is generated by the automorphisms φ, ψ and a fixed element $a \in G$ of the abelian group $(G, +)$ and write $(G, \cdot) = g(G, +, \varphi, \psi, a)$.

We study the problem formulated below.

Problem 1. Let $(Q, \cdot)$ be a $T - quasigroup$. Under which conditions the Q is a quasigroup (of its T - forms $(Q(+), \varphi, \psi, a)$) satisfying the identities P_i of some algebraic structure, where $i = 1, 2, ..., k$?

Professor V. Shcherbacov and his students studied *"Schroder T-quasigroups of generalized associativity"* and *"T-quasigroups with Stein 2-nd and 3-rd identity"* in [12, 13].

We examine the T-quasigroups with Abel-Grassmann identity.

Theorem 3.1. *Let G be a $T-quasigroup$. Then G is $AG-quasigroup$ if and only if for any of its $T-forms$ $(G(+),\varphi,\psi))$, $\varphi^2(x)=\psi(x)$.*

Proof. We rewrite the identity of the $AG-quasigroup$,

$$(xy)\cdot z=(zy)\cdot x, \tag{1}$$

in the following form:

$$\varphi(xy)+\psi(z)=\varphi(zy)+\psi(x). \tag{2}$$

From (2) we have

$$\varphi(\varphi(x)+\psi(y))+\psi(z)=\varphi(\varphi(z)+\psi(y))+\psi(x), \tag{3}$$

$$\varphi^2(x)+\varphi\psi(y)+\psi(z)=\varphi^2(z)+\varphi\psi(y)+\psi(x). \tag{4}$$

If we substitute in equality (4) $x=y=0$, then we obtain

$$\psi(z)=\varphi^2(z). \tag{5}$$

Similarly, if we substitute in equality (4) $z=y=0$, then we obtain

$$\varphi^2(x)=\psi(x). \tag{6}$$

Converse. Substituting the expression $x\cdot y=\varphi(x)+\psi(y)$ in identity (1) then we get (4). Substituting in (4) equalities (5) and (6), $\psi(z)=\varphi^2(z)$ and $\varphi^2(x)=\psi(x)$, we obtain that the identity (4) is true. In this way, in this case, we have that identity (1) is true. The proof is complete. □

Example 3.1. *Examine the group $\mathbb{Z}_n$ of residues modulo n. Define the quasigroup $(G,\cdot)$. We define the binary operation $x\cdot y=3x+9y(mod18)$ for all $x,y\in G$. Then $(G,\cdot)$ is an $AG-quasigroup$. Check. Let $(x\cdot y)\cdot z=(z\cdot y)\cdot x$. Then, $3(3x+9y)+9z=3(3z+9y)+9x(mod18)$, $9x+27y+9z=9z+27y+9x(mod18)$, $0=0(mod18)$.*

Example 3.2. *Denote by $\mathbb{Z}_p=\mathbb{Z}/p\mathbb{Z}=\{0,1,...,p-1\}$ the cyclic group of order p. Let $(G,+)=(\mathbb{Z}_5,+)$, $\varphi(x)=2x$, $\psi(x)=4x$. Then $x\cdot y=2x+4y$ and $\varphi^2(x)=\psi(x)$. Hence, $(G,\cdot)=g(G,+,\varphi,\psi)$ is an AG-quasigroup.*

Below we have constructed the Cayley table for AG-quasigroup $(G, \cdot)$.

$(\cdot)$	0	1	2	3	4
0	0	4	3	2	1
1	2	1	0	4	3
2	4	3	2	1	0
3	1	0	4	3	2
4	3	2	1	0	4

Example 3.3. *Let* $(G, +) = (\mathbb{Z}_5, +)$, $\varphi(x) = 2x$, $\psi(x) = 3x$. *Then* $x \cdot y = 2x + 3y$ *and* $\varphi^2(x) \neq \psi(x)$. *Hence,* $(G, \cdot) = g(G, +, \varphi, \psi)$ *is not an AG-quasigroup.*

Below we have constructed the Cayley table for quasigroup $(G, \cdot)$, *where* $(ab) \cdot c = (cb) \cdot a$ *does not hold in* $(G, \cdot)$. *For example,* $(3 \cdot 4) \cdot 2 \neq (2 \cdot 4) \cdot 3$.

$(\cdot)$	0	1	2	3	4
0	0	3	1	4	2
1	2	0	3	1	4
2	4	2	0	3	1
3	1	4	2	0	3
4	3	1	4	2	0

4. On a method of constructing medial, paramedial and bicommutative topological quasigroups

In this section we examined *Problem*2. In Section 4 we prove a new method of constructing medial, semimedial, paramedial, bicommutative, Manin, Cote and GA non-associative topological quasigroup.

Theorem 4.1. *Let* $(G, +, \tau)$ *be a commutative topological group where G is not a singleton. For* (x_1, y_1) *and* (x_2, y_2) *in* $G \times G$ *define*

$$(x_1, y_1) \circ (x_2, y_2) = (-x_1 - x_2, y_1 + y_2)$$

Then $(G \times G, \circ, \tau_G)$, *relative to the product topology* τ_G, *is a medial, semimedial, paramedial, bicommutative, Manin, Cote and GA non-associative topological quasigroup. Moreover, if* (G, τ) *is* $T_i - space$, *then* $(G \times G, \tau_G)$ *is* $T_i - space$ *too, where* $i = 1, 2, 3, 3.5$.

Proof. **1.** We will prove that $(G \times G, \circ)$ is a quasigroup. To this end, we will show that the equations $y \circ a = b$ and $a \circ x = b$ have unique solutions in $(G \times G, \circ)$. Let $y = (y_1, y_2)$,

$x = (x_1, x_2)$, $a = (a_1, a_2)$ and $b = (b_1, b_2)$. Since, $y \circ a = b$ we have

$$(y_1, y_2) \circ (a_1, a_2) = (b_1, b_2). \tag{7}$$

According to the conditions of the Theorem

$$(y_1, y_2) \circ (a_1, a_2) = (-y_1 - a_1, y_2 + a_2). \tag{8}$$

From (7) and (8) we get

$$-y_1 - a_1 = b_1 \tag{9}$$

and

$$a_2 + y_2 = b_2. \tag{10}$$

From (10) and (9) we obtain

$$y_2 = b_2 - a_2. \tag{11}$$

and

$$y_1 = -a_1 - b_1. \tag{12}$$

Hence, $y_1 = -b_1 - a_1$ and $y_2 = b_2 - a_2$ are solutions of the equation $y \circ a = b$. It is easy to show that any other solutions of that equation coincide with y_1 and y_2.

In this case

$$l((a_1, a_2), (b_1, b_2)) = (-b_1 - a_1, b_2 - a_2)$$

and $l((a_1, a_2), (b_1, b_2)) \circ (a_1, a_2) = (b_1, b_2)$.

We will show that the equation $a \circ x = b$ have unique solutions in $(G \times G, \circ)$. Let $x = (x_1, x_2)$, $a = (a_1, a_2)$ and $b = (b_1, b_2)$. Since $a \circ x = b$ we have

$$(a_1, a_2) \circ (x_1, x_2) = (b_1, b_2). \tag{14}$$

According to the conditions of the Theorem

$$(a_1, a_2) \circ (x_1, x_2) = (-a_1 - x_1, a_2 + x_2). \tag{15}$$

From (14) and (15) we get

$$-x_1 - a_1 = b_1 \tag{16}$$

and

$$a_2 + x_2 = b_2. \tag{17}$$

From (17) and (16) we obtain

$$x_2 = b_2 - a_2. \tag{18}$$

and

$$x_1 = -a_1 - b_1. \tag{19}$$

Hence, $x_1 = -b_1 - a_1$ and $x_2 = b_2 - a_2$ are solutions of the equation $a \circ x = b$. It is easy to show that any other solutions of that equation coincide with x_1 and x_2. Then $x_1 = -b_1 - a_1$ and $x_2 = b_2 - a_2$ are unique solutions.

In this case

$$r((a_1, a_2), (b_1, b_2)) = (-b_1 - a_1, b_2 - a_2)$$

and $(a_1, a_2) \circ r((a_1, a_2), (b_1, b_2)) = (b_1, b_2)$.

Thus $(G \times G, \circ)$ is a quasigroup.

2. We will prove that associativity

$$((x_1, y_1) \circ (x_2, y_2)) \circ (x_3, y_3) = (x_1, y_1) \circ ((x_2, y_2) \circ (x_3, y_3)) \tag{20}$$

does not hold in $(G \times G, \circ)$.

Indeed, for the first side of the law (20) we obtain

$$((x_1, y_1) \circ (x_2, y_2)) \circ (x_3, y_3) = (-x_1 - x_2, y_1 + y_2) \circ (x_3, y_3) =$$

$$= (x_1 + x_2 - x_3, y_1 + y_2 + y_3). \tag{21}$$

Similarly, for the second side of the law (20) we have

$$(x_1, y_1) \circ ((x_2, y_2) \circ (x_3, y_3)) = (x_1, y_1) \circ (-x_2 - x_3, y_2 + y_3) =$$

$$= (-x_1 + x_2 + x_3, y_1 + y_2 + y_3). \tag{22}$$

From (21) and (22) it is clear that associativity does not hold in $(G \times G, \circ)$.

3. We will show that $(G \times G, \circ)$ is a medial quasigroup that is, the property $xy \cdot zt = = xz \cdot yt$ holds.

Let $x = (x_1, y_1), y = (x_2, y_2), z = (x_3, y_3), t = (x_4, y_4)$, then

$$((x_1, y_1) \circ (x_2, y_2)) \circ ((x_3, y_3) \circ (x_4, y_4)) = ((x_1, y_1) \circ (x_3, y_3)) \circ ((x_2, y_2) \circ (x_4, y_7)). \tag{23}$$

According to the Theorem for the first side of the law (23) we have

$$((x_1, y_1) \circ (x_2, y_2)) \circ ((x_3, y_3) \circ (x_4, y_4)) =$$

$$= ((-x_1 - x_2, y_1 + y_2)) \circ ((-x_3 - x_4, y_3 + y_4)) =$$

$$= (x_1 + x_2 + x_3 + x_4, y_1 + y_2 + y_3 + y_4). \tag{24}$$

Similarly, for the other side of the law (23) we get

$$((x_1, y_1) \circ (x_3, y_3)) \circ ((x_2, y_2) \circ (x_4, y_4)) =$$

$$= (-x_1 - x_3, y_1 + y_3) \circ (-x_2 - x_4, y_2 + y_4) =$$

$$= (x_1 + x_3 + x_2 + x_4, y_1 + y_3 + y_2 + y_4). \tag{25}$$

From (24) and (25) we obtain that both sides are equal and $(G \times G, \circ)$ is a medial quasigroup. Similarly, it is shown that paramediality and bicommutative does hold in $(G \times G, \circ)$.

4. We will show that $(G \times G, \circ)$ is Manin quasigroup that is, the property $x(y \cdot xz) = (xx \cdot y)z$ holds.

Let $x = (x_1, y_1), y = (x_2, y_2), z = (x_3, y_3)$ then

$$(x_1, y_1)\circ((x_2, y_2)\circ((x_1, y_1)\circ(x_3, y_3))) = (((x_1, y_1)\circ(x_1, y_1))\circ(x_2, y_2))\circ(x_3, y_3)). \tag{26}$$

According to the Theorem for the first side of the law (26) we have

$$(x_1, y_1) \circ ((x_2, y_2) \circ ((x_1, y_1) \circ (x_3, y_3))) =$$

$$(x_1, y_1) \circ ((x_2, y_2) \circ (-x_1 - x_3, y_1 + y_3)) =$$

$$(x_1, y_1) \circ (-x_2 + x_1 + x_3, y_1 + y_2 + y_3) =$$

$$(-x_1 + x_2 - x_1 - x_3, 2y_1 + y_2 + y_3). \tag{27}$$

Similarly, for the other side of the law (26) we get

$$(((x_1, y_1) \circ (x_1, y_1)) \circ (x_2, y_2)) \circ (x_3, y_3)) =$$

$$((-x_1 - x_1, y_1 + y_1) \circ (x_2, y_2)) \circ (x_3, y_3)) =$$

$$(+x_1 + x_1 - x_2, y_1 + y_1 + y_2) \circ (x_3, y_3) =$$

$$= (-x_1 - x_1 + x_2 - x_3, 2y_1 + y_2 + y_3). \tag{28}$$

From (27) and (28) we obtain that both sides are equal and $(G \times G, \circ)$ is a Manin quasigroup. Similarly, it is shown that $(G \times G, \circ)$ is a Cote quasigroup.

5. Similarly, we will show that GA identity is fulfilled in $(G \times G, \circ)$. Let $x = (x_1, y_1), y = (x_2, y_2), z = (x_3, y_3)$ then

$$((x_1, y_1) \circ (x_2, y_2)) \circ ((x_3, y_3) = (x_3, y_3) \circ ((x_2, y_2)) \circ (x_1, y_1)) \tag{29}$$

Indeed, according to the Theorem for the first side of the law (29) we have

$$((x_1, y_1) \circ (x_2, y_2)) \circ (x_3, y_3) =$$

$$(-x_1 - x_2, y_1 + y_2) \circ (x_3, y_3) =$$

$$(x_1 + x_2 - x_3, y_1 + y_2 + y_3). \tag{30}$$

Similarly, for the other side of the law (29) we get

$$(x_3, y_3) \circ ((x_2, y_2)) \circ (x_1, y_1)) =$$

$$(x_3, y_3) \circ (-x_2 - x_1, y_2 + y_1) =$$

$$(-x_3 + x_2 + x_1, y_3 + y_2 + y_1). \tag{31}$$

From (30) and (31) we obtain that both sides are equal and $(G \times G, \circ)$ is a GA quasigroup.

Multiplication $(\circ)$ and divisions $l(a, b)$ and $r(a, b)$ are jointly continuous relative to the product topology.

Consequently, $(G \times G, \circ, \tau_G)$ is a non-associative, medial, semimedial, paramedial, bicommutative, Manin, Cote and GA topological quasigroup.

If (G, τ) is T_i-space, then according to Theorem 2.3.11 in [6], a product of T_i-spaces is a T_i-spaces, where $i = 1, 2, 3, 3.5$. The proof is complete. □

In [1] was proved the following Theorem.

Theorem 4.2. *Let $(G, +, \tau)$ be a commutative topological group where G is not a singleton. For (x_1, y_1) and (x_2, y_2) in $G \times G$ define*

$$(x_1, y_1) \circ (x_2, y_2) = (x_1 + y_1 - x_2, x_2 + y_2 - y_1).$$

Then $(G \times G, \circ, \tau_G)$, relative to the product topology τ_G, is a paramedial, non-medial and non-associative topological quasigroup. Moreover, if (G, τ) is $T_i - space$, then $(G \times G, \tau_G)$ is $T_i - space$ too, where $i = 1, 2, 3, 3.5$.

The following Theorem was proved in [2].

Theorem 4.3. *Let $(G, +, \tau)$ be a commutative topological group where G is not a singleton. For (x_1, y_1) and (x_2, y_2) in $G \times G$ define*

$$(x_1, y_1) \circ (x_2, y_2) = (-x_1 - y_1 + y_2, -x_2 - y_2 + x_1).$$

Then $(G \times G, \circ, \tau_G)$, relative to the product topology τ_G, is a non-associative, medial, AG and AD-topological quasigroup. Moreover, if (G, τ) is $T_i - space$, then $(G \times G, \tau_G)$ is $T_i - space$ too, where $i = 1, 2, 3, 3.5$.

Example 4.1. *Let $G = \{0, 1, 2\}$. We define the binary operation "+".*

(+)	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

Then $(G, +)$ is a commutative group. Define a binary operation $(\circ)$ on the set $G \times G$ by $(x_1, y_1) \circ (x_2, y_2) = (-x_1 - x_2, y_1 + y_2)$ for all $x_1, y_1, x_2, y_2 \in G \times G$. If we label the elements as follows $(0, 0) \leftrightarrow 0$, $(0, 1) \leftrightarrow 1$, $(0, 2) \leftrightarrow 2$, $(1, 0) \leftrightarrow 3$, $(1, 1) \leftrightarrow 4$, $(1, 2) \leftrightarrow 5$, $(2, 0) \leftrightarrow 6$, $(2, 1) \leftrightarrow 7$, $(2, 2) \leftrightarrow 8$, then obtain:

$(\circ)$	0	1	2	3	4	5	6	7	8
0	0	1	2	6	7	8	3	4	5
1	1	2	0	7	8	6	4	5	3
2	2	0	1	8	6	7	5	3	4
3	6	7	8	3	4	5	0	1	2
4	7	8	6	4	5	3	1	2	0
5	8	6	7	5	3	4	2	0	1
6	3	4	5	0	1	2	6	7	8
7	4	5	3	1	2	0	7	8	6
8	5	3	4	2	0	1	8	6	7

Then $(G \times G, \circ)$ is a medial, semimedial, paramedial, bicommutative, Manin, Cote, GA non-associative quasigroup.

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