ABOUT APPLICATIONS OF TOPOLOGICAL STRUCTURES IN COMPUTER SCIENCES AND COMMUNICATIONS

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Abstract. One of the central problems in computer science and, in particular, in programming is the correctness problem which contains:

- the question of whether a program computes a given function;
- the problem to decide whether an element of the space is equal to a fixed element;
- whether two elements of a given space are equal and whether one approximates the other in the specialization order.

In this article is examined the role of quasi-metrics and of Alexandroff spaces in the solving of some problems in the design of systems of the computer science an communications.
Keywords: Alexandroff space, distance, digital space, quasimetric.

APLICAŢIILE STRUCTURILOR TOPOLOGICE ÎN INFORMATICĂ ŞI COMUNICAŢII

Rezumat. Una din problemele centrale ale informaticii şi, în special, ale programării este problema corectitudinii ce conţine:

- problema dacă un program calculează o funcţie dată;
- problema pentru a decide dacă un punct al spaţiului coincide cu un alt punct fixat;
- dacă doua puncte dintr-un spaţiu dat coincid şi dacă unul se apropie (formând o secvenţă) de celalalt în careva ordine specificată.

În acest articol se examinează rolul cvasi-metricilor şi al spaţiilor Alexandroff la rezolvarea unor probleme legate de proiectarea sistemelor din domeniile informaticii şi comunicaţiei.
Cuvinte chee: spaţiu Alexandroff, distanţă, spaţiu digital, quasi-metrică.

1. Introduction

The dynamic transition of our technological civilization to digital processing and data transmission systems created many problems in the design of modern systems in computer science and telecommunications. Providing robustness and noise immunity is one of the most important and difficult tasks in data transmission, recording, playback, and storage. The distance between information plays a paramount role in mathematics, computer science, and other interdisciplinary research areas. The first among many scientists in the field, who presented the theoretical solutions to error detection and error correction problems, were C. Shannon, R. Hamming, and V. Levenshtein (see [44, 45, 23, 29]).

We begin this section with introductions into the field, focusing mainly on abstract monoid of strings $L(A)$.

A monoid is a semigroup with an identity element.

Fix a non-empty set A. The set A is called an *alphabet*. Let $L_0(A)$ be the set of all finite strings $a_1a_2\dots a_n$ with $a_1, a_2, \dots, a_n \in A$. Let ε be the empty string and $L(A) = L_0(A) \cup \{\varepsilon\}$. Consider the strings $a_1a_2\dots a_n$ such that $a_i = \varepsilon$ for some $i \leq n$. We consider that $n = 1$ if each $a_i = \varepsilon$. Denote by $L^*(A)$ the set of such strings.

If $a_i \neq \varepsilon$, for any $i \leq n$ or $n = 1$ and $a_1 = \varepsilon$, the string $a_1 a_2 \dots a_n$ is called a *canonical string* or a *word*. Hence $L(A) \subseteq L^*(A)$ and $L(A)$ is the set of all canonical strings. We consider that

The set

$$Sup(a_1 a_2 \dots a_n) = \{a_1, a_2, \dots, a_n\} \cap A$$

is the support of the string $a_1 a_2 \dots a_n$,

$$l^*(a_1 \dots a_n) = max\{i : a_i \neq \varepsilon\}$$

is the duration of the string $a_1 a_2 \dots a_n$ and

$$l(a_1 \dots a_n) = |Sup(a_1 \dots a_n)|$$

is the length of the string $a_1 a_2 \dots a_n$.

For any string $a_1 a_2 \dots a_n$ we have $l(a_1 \dots a_n) \leq l^*(a_1 \dots a_n)$. By definition, $l(\varepsilon) = l^*(\varepsilon) = 0$.

For two strings $a_1 \dots a_n$ and $b_1 \dots b_m$, their product(concatenation) is $a_1 \dots a_n b_1 \dots b_m$. We put $\varepsilon \cdot a = a \cdot \varepsilon = a$ for any $a \in L^*(A)$. If $a = a_1 a_2 ... a_n \in L^*(A)$ and $n \geq 2$, then $\Psi(b) = a_1 \cdot a_2 \cdot ... \cdot a_n$ is the canonical word equivalent with the string a. If $\Psi(a) = \Psi(b)$,then the strings a, b are called equivalent. In this case any string is equivalent to one unique canonical string.

The sets $L(A)$ and $L^*(A)$ become the monoids with identity ε. The mapping $\Psi : L^*(A) \longrightarrow L(A)$ is a homomorphism. Let G be a finite set of generators of a monoid M. In this case unity is an element of G. If M is a group,then for convenience we suppose that the inverse of a generator is a generator. Any word $a = a_1 a_2 ... a_n \in L^*(G)$ determine the element $g(a) = a_1 \cdot a_2 \cdot ... \cdot a_n \in M$ and $g : L^*(G) \longrightarrow M$ is a homomorphism of $L^*(G)$ onto M. A monoid M is said to have a solvable word problem with respect to the set of generators G if one can effectively determine whether or not two words $a, b \in L^*(G)$ represent the same element in M. A monoid (group) M is said to be computable if it has a recursive realization $\{M, \xi\}$ - i.e. it is isomorphic to the monoid formed by a recursive subset S of the positive integers and a recursive function $\xi(i, j)$ on S that satisfies the monoid (group) multiplication axioms. The monoid $L^*(A)$ is computable and has a solvable word problem.

M. O. Rabin [36] has proved that a finitely generated group (monoid) has a solvable word problem (with respect to a given system of generators) if and only if it is computable. In 1947 Post [34, 35] showed the word problem for semigroups to be undecidable. This result was strengthened in 1950 by Turing [51], who showed the word problem to be undecidable for cancellation semigroups,i.e. semigroups satisfying the cancellation property: if $xy = xz$ or $yx = zx$, then $y = z$.

In 1966 Gurevich [22] showed the word problem to be undecidable for finite semigroups. In [50] by him were proved:

(G1) The undecidability of the word problem for the finite monoid A with the generators $\{a, h, , d, \}$ and identical correlations: $ac = ca$, $ad = da$, $bc = cb$, $bd = db$, $eca = ce$, $edb = de$, $cca = ccae$;

(G2) The undecidability of the word problem for the finite monoid B with the generators $\{a, h, , d, \}$ and identical correlations: $ac = ca$, $ad = da$, $bc = cb$, $bd = db$, $eca = ce$, $edb = de$, $cdca = cdcae$, $caaa = aaa$, $daaa = aaa$. In the monoid is undecidable the word problem for the strings equivalent with the word .

Throughout the history of the subject, computations in groups have been carried out using various normal forms. These usually implicitly solve the word problem for the groups in question. In 1911 Max Dehn [11] proposed that the word problem was an important area of study in its own right together with the conjugacy problem and the group isomorphism problem. In 1912 he gave an algorithm that solves both the word and conjugacy problem for the fundamental groups of closed orientable two-dimensional manifolds of genus greater than or equal to 2 [12]. Subsequent authors have greatly extended Dehn's algorithm and applied it to a wide range of group theoretic decision problems [2, 7, 30, 49].

It was shown by Pyotr Novikov [32] in 1955 that there exists a finitely presented group G such that the word problem for G is undecidable. A different proof was obtained by William Boone in 1958 (see [2, 6, 7, 32]).

One of the central problems in computer science and, in particular, in programming is the correctness problem which contains:

- the question of whether a program computes a given function;
- the problem to decide whether an element of the space is equal to a fixed element;
- whether two elements of a given space are equal and whether one approximates the other in the specialization order.As a rule, a database system maintain items in the form of sequences of special type.

The similarity search process is obtained by defining a similarity function. In many applications a distance function can be easily and more intuitively defined than a similarity function. Moreover,it easy to obtain a similarity function by given a distance function and vice versa. For that we apply the principle: the smaller the distance the higher the similarity.

2. Alexandroff spaces

For a topological space X and the points $a, b \in X$ we put $O(a) = \cap\{U \subset X : x \in U, U$ *is open in* $X\}$ and $a \preceq b$ if and only if $a \in cl_X\{b\}$. Then $\preceq$ is a preordering on X. A binary relation $\preceq$ on a space X is a a preorder, or quasiorder, if it is reflexive and transitive, i.e., for all a, b and c in X, we have that:

- $a \preceq a$ (reflexivity);
- if $a \preceq b$ and $b \preceq c$, then $a \preceq c$(transitivity).

If X is a T_0-space, then $\preceq$ is an ordering on X: if $a \preceq\preceq b$ and $b \preceq a$, then $a = b$ (anti-symmetry).

A topological space X is called a pseudo-discrete space if the intersection of any family of open sets is open. By definition, the space X is a pseudo-discrete space if and only if the sets $O(x)$, $x \in X$, are open in X. A topological space X is called an Alexandroff space if it is a pseudo-discrete T_0-space [3, 4]. A connected Alexandroff space is called a topological digital space.

Quasi-metric [31] on a set X we call a function $d : X \times X \longrightarrow R$ with the properties:

(M1): $d(x, x) = 0$ and $d(x, y) \geq 0$ for all $x, y \in X$;

(M2): $d(x, y) + d(y, x) = 0$ if and only if $x = y$;

(M3): $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in X$.

If $d(x, y) = d(y, x)$ for all $x, y \in X$, then the quasi-metric d is called a metric.

A function d with the properties (M1) and (M2) is called a distance on a set X. A function d with the property (M1) is called a pseudo-distance on a set X. A function d with the properties (M1) and (M3) is called a pseudo-quasi-metric on a set X.

Let d be a pseudo-distance on X and $B(x, d, r) = \{y \in X : d(x, y) < r\}$ be the ball with the center x and radius $r > 0$. The set $U \subset X$ is called d-open if for any $x \in U$ there exists $r > 0$ such that $B(x, d, r) \subset U$. The family $T(d)$ of all d-open subsets is the topology on X generated by d. A pseudo-distance space is a sequential space, i.e. a set $B \subset X$ is closed if and only if together with any sequence it contains all its limits [15].

If d is a quasi-metric,then $T(d)$ is a T_0-topology. For any distance that statement is not true.

Example 2.1. Let $n \geq 3$ and $X = \{a_1, a_2, ..., a_n\}$. We put $d(x, x) = 0$ for any $x \in X$, $d(a_i, a_j) = 1$ for $j < i$, $d(a_i, a_{i+1}) = 0$ for any $i < n$ and $d(a_i, a_j = 1$ for $i < i + 2 \leq j \leq n$. In the topology $T(d)$ we have $O(a_i) = \{a_j : i \leq j \leq n\}$ for each $i \leq n$. The family $\{O(x) : x \in X\}$ is a base of the T_0-topology $T(d)$.

Example 2.2. Let $X = \{a, b, c\}$. We put $d(x, x) = 0$ for any $x \in X$, $d(a, b) = d(b, c) = d(c, a) = 0$ and $d(a, c = d(c, b) = d(b, a) = 1$. In the topology $T(d)$ we have $O(a) = X$ for each $x \in X$ and $T(d) = \{\emptyset, X\}$ is the anti-discrete topology.

The pseudo-distance d is discrete if there exist a number $c = c(d) > 0$ such that for any two distinct points $x, y \in X$ we have or $d(x, y) = 0$ or $d(x, y) \geq c$ [31, 8, 9]. If d is a discrete quasi-metric on X, then $O(a) = B(a, d, c(d))$ for any point $a \in X$ and the space $(X, T(d))$ is an Alexandroff space. The pseudo-distance is an integer pseudo-distance, if $d(x, y) \in \{0, 1, 2, ...\}$ for any $x, y \in X$. The integer pseudo-distance is discrete with $c(d) = 1$.

If $\preceq$ is a preordering on a set X, then we define two pseudo-quasi-metrics d_l and d_r on X, where:

- $d_l(x, y) = d_r(y, x)$ for any $x, y \in X$ with $y \preceq x$ and $x \preceq y$;
- for $x \preceq y$ and $x \not\succeq y$ we put $d_l(x, y) = 1$, $d_l(y, x) = 0$, $d_r(x, y) = 0$, $d_r(y, x) = 1$;
- if $x \not\preceq y$ and $y \not\preceq X$, then $d_l(x, y) = d_r(x, y) = 1$.

In this case $d_s(x, y) = d_r(x, y) + d_l(x, y)$ is a pseudo-metric. In general, a sum of quasi-metrics is also a quasi-metric, and may not be a metric. If $\preceq$ is an ordering, then d_r, d_l arequasi-metrics and d_s a metric.

For any points $a, b \in X$ we put $(-, a] = \{y \in X : y \preceq a\}$, $[a, +) = \{y \in X : a \preceq y\}$ and $[a, b] = \{y \in X : y \preceq b\} \cap \{y \in X : a \preceq y\}$. The set $L = [a, +)$ is called an ω-set and $o_L = a$.

For any point $a \in X$ we have $B(x, d_l, r) = (-, a]$ and $B(x, d_l, r) = [a, +)$ for any $r \in (0, 1]$. We say that $T(\preceq) = T(d_r))$ is the topology induced by the pre-ordering $\preceq$.

Fix a space X. As in [17, 19, 18] we say that V is an f-set if V is open and there exists a point $o_V \in V$ such that $V = [o_V, +)$. Any f-set is an ω-set.

If X is an Alexandroff space, then any set $V = O(x)$ is an f-set with $o_V = x$.

From the above reasoning follows:

Theorem 2.3. *For a topological space X the following assertions are equivalent:*

1. X is a pseudo-discrete space.

2. Any ω-set is an f-set of X.

3. The topology of X is induced by some preordering.

4. The topology of X is generated by some discrete pseudo-quasi-metric.

5. The topology of X is generated by some integer pseudo-quasi-metric.

Corollary 2.4. *For a topological space X the following assertions are equivalent:*

1. X is an Alexandroff space.

2. X is a T_0-space and any ω-set is an f-set of X.

3. The topology of X is induced by some ordering.

4. The topology of X is generated by some discrete quasi-metric.

5. The topology of X is generated by some integer quasi-metric.

3. Spaces and lattices

An ordered set X is called:

- an upper semi-lattices if for any two elements $x, y \in X$ is determined the greatest lower bound $x \vee y$;

- a lower semi-lattices if for any two elements $x, y \in X$ is determined the least upper bound $x \wedge y$;

- a lattices if for any two elements $x, y \in X$ are determined the greatest lower $x \vee y$ and the least upper $x \wedge y$ bounds.

Proposition 3.1. *For an ordered set X the following assertions are equivalent:*

1. X is an upper semi-lattice.

2. Intersection of two ω-sets is an ω-set.

Proof. Let U and V be two ω-sets of X and $W = U \cap V$. If W is an ω-set, then $o_W = o_U \vee o_V$. Inversely, if $a = o_U \vee o_V$, then W is an ω-set with $o_W = a$. The proof is complete.

Proposition 3.2. *For an ordered set $(X, \leq)$ the following assertions are equivalent:*

1. $(X, \leq)$ is a linearly ordered set.

2. Union of two ω-sets is an ω-set.

Proof. Let U and V be two f-sets of X and $W = U \cup V$. If $\leq$ is a linear ordering, then or $U \subset V$, or $V \subset U$ and W is an f-set. Assume that W is an f-set. We have two cases:

Case 1. $o_W \in \{o_U, o_V\}$.

If $o_W = o_U$, then $V \subseteq U$ and $o_U \leq o_V$. In this case $\leq$ is a linear ordering.

Case 2. $o_W \notin \{o_U, o_V\}$.

In this case $o_W \in W$ and $o_W \notin U \cup V = W$, a contradiction. This case is impossible. The proof is complete.

Let X be a T_0-space and $\leq$ be the ordering induced by the topology of X: $x \leq y$ if and only if $x \in cl\{y\}$. A space X is called an f-space if the f-sets form an open base of X and the intersection of two f-sets is empty or an f-set [17]. If X is an f-space, then $X_0 = \{x \in X : [x, +) \textit{ is open}\}$ is the base of X. The base X_0 is dense in X and the weight $w(X) = |X_0|$.

A discrete space is an f-space and the f-sets are singleton sets.

Assume that $X_0 \subset X$. As in [18], the pair $(X_0, \leq)$ is called a subsaile of the space X if for any two points $x, y \in X_0$ for which $[x, +) \cap [y, +) \neq \emptyset$ there exists the upper bound

$x \vee y \in X_0$. The base X_0 of the f-space X is a subsaile of the space X [17, 18].

If $[x,+) \cap [y,+) \neq \emptyset$ ($(-,x] \cap (-,y] \neq \emptyset$), then we say that the points x, y are upper (lower) compatible. If $y \in Int_X[x,+)$, then we put $x \prec y$. By definition, from $x \prec y$ it follows that $x \leq y$. If X is an Alexandroff space, then from $x \leq y$ it follows that $x \prec y$.

An Alexandroff space X is an f-space if and only if for any two upper compatible points $x, y \in X$ there exists $x \vee y \in X$. If an f-space X is an Alexandroff space, then the base of X is X.

Now we mention the following simple relations (see [18]):

(a) if $x \leq y$ and $y \prec z$, then $x \prec z$;

(b) if $x \prec y$ and $y \leq z$, then $x \prec z$;

(c) if $x \leq u \leq z$, $y \leq u \leq z$, $x \prec z$ and $y \prec z$, then $u \prec z$;

(d) if $x_1 \prec y_1 \leq y$, $x_2 \prec y_2 \leq y$ and $x = x_1 \vee x_2$, then $x \prec y$;

(e) if $g : X \longrightarrow Y$ is a continuous mapping, $x, y \in X$ and $x \leq y$, then $g(x) \leq g(y)$.

For the concept of the f-space there exists algebraical (abstract) description [17, 18]. An f-space is a triplet $(X, X_0, \leq)$, where:

- X_0 is a non-empty subset of X;
- $\leq$ is a partial ordering on X;
- if the elements $x, y \in X_0$ are upper compatible, then in X_0 there exists $x \vee y$;
- if $x, y \in X$, $y \not\leq x$, then there exists $z \in X_0$ such that $z \leq y$ and $z \not\leq x$;
- for any $x \in X$ there exists $y \in X_0$ such that $y \leq x$.

A space X is called an A-space if X is a T_0-space and there exists a subsaile $(X_0, \leq)$ for which:

(A1) the family $\{Int_X[x,+) : x \in X_0\}$ is an open base of the space X;

(A2) if $x \in X_0$, $y \in X$ and $x \prec y$, then there exists $z \in X_0$ such that $x \prec z \prec y$.

The subspace X_0 is called a base of the A-space X.

The following two examples are simple, but reflect clearly the structure of A-spaces and f-spaces.

Example 3.3. Let X be the closed interval $[0,1]$ with the topology $T = \{\emptyset, X\} \cup \{(t,1] : t \in X\}$, where $(t,1]$ is semi-open interval $\{x : t < x \leq 1\}$. The space X is a compact T_0-space and the topological ordering on X coincide with the usual linear order on the numerical interval. We have $[0,+) = Int_X[0,+) = X$ and $[x,+) = [x,1)$, $Int_X[x,+) = (x,1]$ for any $x > 0$. The space X is an A-space. If X_0 is dense in X and $0 \in X_0$, then X_0 is a base of X.

Example 3.4. Let X be the space of reals Y be a subset of X and on X we consider the topology $T = \{\emptyset, X\} \cup \{(t,+\infty) : t \in X\} \cup \{[t,+\infty) : t \in Y\}$. The space X is a T_0-space and the topological ordering on X coincide with the usual linear order on the space of real numbers. We have $[y,+) = Int_X[y,+)$ $[x,+\infty)$ for $y \in Y$ and $[x,+) = [x,+\infty)$, $Int_X[x,+)$ $= (x,+\infty)$ for any $x \in X \setminus Y$. The space X is an A-space. If X_0 is a dense subset of the reals and $Y \subset X_0$, then X_0 is a base. If Y is dense in the space of reals, then Y is a base and X is an f-space. If Y is not dense in the space of reals, then X is not an f-space. If $Y = X$, then X is the unique base of X and X is an Alexandroff space too. For $Y \neq X$ the space X is not an Alexandroff space.

Proposition 3.5. *Let X_0 be the base of an A-space X. If $a \in X$ is a minimal element of the ordered set $(X, \leq)$, then $a \in X_0$ and $a \in Int_X[a,+) \subset U$ provided U is open in X and $a \in U$.*

Proof. Since a is a minimal element, $a \in [x,+)$ if and only if $a = x$. Let U be an open subset of X and $a \in U$. then there exists $x \in X_0$ such that $a \in Int_X[x,+) \subset U$. Thus $a = x$, $a \in X_0$ and $a \in Int_X[a,+) \subset U$. The proof is complete.

The following assertion for T_1-spaces was proved in [18].

Proposition 3.6. *For any T_0-space X there exists an f-space X_f which contains X as a dense subspace and weight $w(X_f = w(X)$.*

Proof. Let T be the topology of the space X. Fix an open base $\mathcal{B}$ of the space X. Suppose that $|\mathcal{B}| = w(X)$. We assume that $U \cap V \in \mathcal{B}$ for all $U, V \in \mathcal{B}$ with $U \cap V \neq \emptyset$. If $U \in \mathcal{B}$ is an open f-set, then is determined the point o_U. We put $B_f = \{U \in \mathcal{B} : U \textit{ is an } f\textit{-set}\}$ and $B_+ = \mathcal{B} \setminus B_f$. Let $X_0^f = \{o_U : U \in B_f\}$. Fix a set $X_0^+ = \{o_U : U \in B_+\}$ for which $X_0^+ \cap X = \emptyset$ and the correspondence $o_U \to U$ is a one-to-one mapping of X_0^+ onto B_+. Now we put $X_f = X \cup X_0^+$, $X_0 = X_0^f \cup X_0^+$ and $U^+ = U \cup \{o_V : V \subset U, V \in \mathcal{B}\}$ for each $U \in \mathcal{B}$. If $U, V\mathcal{B}$ and the set $W = U \cap V$ is non-empty, then $W \in \mathcal{B}$, $W^+ = U^+ \cap V^+$ and $U^+ \cap X = U$. Hence $\mathcal{B}^+ = \{U^+ : U \in \mathcal{B}\}$ is an open base of some topology T_f on X_f and (X, T) is a dense subspace of the space (X_f, T_f). The ordering $x \leq y$ on X_f is the following:

- if $x, y \in X$, then the relation between x, y in X_f is as in X;
- if $x = o_U \in X_f \setminus X$ and $y \in X$, then $y \not\leq x$ and $x \leq y$ if and only if $y \in U$;
- if $x = o_U \in X_f \setminus X$ and $y = o_V \in X_f \setminus X$, then $x \leq y$ if and only if $V \subset U$.

In this case for any $U \in \mathcal{B}$ the set U^+ is an f-set and $U^+ = [o_U, +)$. By construction, X_f is an T_0-space. Hence X_f is an f-space and an A-space with the base X_0. The proof is complete.

A space X is called an A_0-space if X is an A-space with the the lower bound, i.e. there exists a point $a \in X$ such that $[a,+) = X$. In this case X is an f-set and $o_X = a$.

If $X \in \mathcal{B}$, then o_X is the the lower bound of the space X_f.

Hence, from the proof of Proposition 3.6 it follows:

Proposition 3.7. *For any T_0-space X there exists an A_0-space X^f which contains X as a dense subspace and weight $w(X^f = w(X)$. Moreover, if X is an A-space, then X is a dense subspace of an A_0-space X^f for which $|X^f \setminus X| = 1$.*

Example 3.8. Let X be an infinite non-discrete T_1-space. Then X is not an A-space. By virtue of Proposition 3.6 X is a dense subspace of the f-space X_f. Therefore the subspace of an f-space is nt obligatory A-space.

We mention that a closed subspace of an A-space is an A-space. Similarly, an open subspace of an A-space is an A-space.

An ordered set X is directed is any two elements $x, y \in X$ are upper compatible, i. e. there exists $z \in X$ such that $xx \leq z$ and $y \leq z$.

In [18] was proposed the following useful construction. Let X be an A-space with the base X_0. Denote by $\Pi(X_0)$ the family of all subsets S of X_0 with the following properties:

- S is a non-empty directed subset of X_0;

- for any $x \in S$ there exists $y \in S$ such that $x \prec y$;

- if $x \in S$, $z \in X_0$ and $z \leq x$, then $z \in S$.

The elements of $\Pi(X_0$ are called s-points. If $x \in X$, then $\eta(x) = (-, x] \cap X_0$ is an s-point. The mapping $\eta : X \longrightarrow \Pi(X_0)$ is a mapping of X into $\Pi(X_0)$. For any $x \in X_0$ we put $\theta(x) = \{S \in \Pi(X_0) : x \in S\}$. The family $\{\theta(x) : x \in X_0\}$ is an open base of the topology on $\Pi(X_0)$. In this case $\eta : X \longrightarrow \Pi(X_0)$ is a topological embedding.

A space X is called a complete A-space if X is an A-space and for any base X_0 of X the mapping $\eta : X \longrightarrow \Pi(X_0)$ is a homeomorphism of X onto $\Pi(X_0)$.

By virtue of Theorem 1 from [18], an A-space X is a complete A-space if and only if for each A-space Y, any base Y_0 of Y and every continuous mapping $\varphi : Y_0 \longrightarrow X$ there exists a continuous mapping $\psi : Y \longrightarrow X$ such that $\varphi = \psi | Y_0$.

Any T_0-space X can be embedded in a T_0-space X_b with the upper bound and $|X_b \setminus X| \leq 1$. If $X \neq (-, x]$ for any point $x \in X$, then fix a point $b \notin X$ and put $X_b = X \cup \{b\}$ and with the topology $T_b = \{\emptyset\} \cup \{U \cup \{b\} : U \in T\}$, where T is the topology of X. In this case the set $\{b\}$ is open and dense in X_b.

An injective space is a complete A_0-space with the upper bound [38, 41, 18].

The A-spaces and injective spaces were introduced by Dana Scott [38, 41] and Yurii L. Ershov [17, 19, 18] with the aims:

- of the construction of a model for Lambda calculus Alonzo Church [38, 41];

- of the analysis of the concept "data types" [38, 41];

- of the investigation the semantics of programming languages [38, 41];

- of the study of computable functionals [17, 19, 18].

If X is a complete A-space, then any non-empty upper directed set have the greatest lower bound (supremum) [18]. From this assertion immediately follows:

- in a complete A_0-space, then any non-empty set have the least upper bound (infimum) [18];

- an injective space is a complete lattice (any non-empty set have the least upper and the the greatest lower bounds) [38, 41].

The following theorem affirmatively solve one Yu. L. Ershov's question ([18], p. 396).

Proposition 3.9. *Assume that Y is a retract of an A-space X. Then Y is an A-space. Moreover, if X is an A_0-space, then Y is an A_0-space too.*

Proof. Let X_0 be the base of the space X.

For the concept of the A-space there exists algebraical (abstract) description. An A-space is a quadruple $(X, X_0, \leq, \prec)$, where:

- $\leq$ is a partial ordering on X;

- X_0 is a subsaile of X;

- $\prec$ is a binary relation on X;

- if $x, y \in X$ and $x \prec y$, then $x \leq y$;

- if $x, y, z \in X$, $x \prec y$ and $y \leq z$, then $x \prec z$;

- if $x, y, z \in X$, $x \leq y$ and $y \prec z$, then $x \prec z$;

- if $x, y \in X_0$, $a \in X$, $x \prec a$ and $y \prec a$, then $x \vee y \in X_0$ and $x \vee y \prec a$;

- if $x, y \in X$, $y \not\leq x$, then there exists $z \in X_0$ such that $z \prec y$ and $z \not\leq x$;

- for any $x \in X$ there exists $y \in X_0$ such that $y \prec x$.

If $(X, X_0, \leq, \prec)$ is an abstract A-space, then $\{U(x) = \{y \in X : x \prec y\} : x \in X_0\}$ is the open base of the topology on X and X_0 is the base of X.

That permit to evolve the abstract concept of "approximation" of the elements by the elements with the "visible" properties. Let $(X, X_0, \leq, \prec)$ be an abstract A-space. If $x, y \in X$ and $x \leq y$, then x is called an "approximation" of y [38, 41]. If $x \in X_0$, $y \in X$ and $x \leq y$, then $x$is an "best approximation" of y. If $x \prec y$ and $x \in X_0$, then x is a "recognized approximation" of y [18].

4. Spaces of strings

Fix a non-empty T_0-space A with a fixed point ε such that $\varepsilon \leq x$ for each point $x \in X$. On $\mathbb{N}$ consider the discrete topology. Denote by $C(\mathbb{N}, A)$ the family of all mappings $s : \mathbb{N} \longrightarrow A$ in the topology of pointwise convergence. The elements of $C(\mathbb{N}, A)$ are called sequences.

Consider that d is a quasi-metric on A. For any two sequences $a, b : \mathbb{N} \longrightarrow A$ we determine the distance $d^*(a, b) = \Sigma\{d(a(i), b(i)) :\in \mathbb{N}\}$. For some $a, b \in C(\mathbb{N}, A)$ we have $d^*(a, b) = \infty$. The distance d^* has the properties:

- $d^*(a, b) \geq 0$;

- $d^*(a, b) + d^*(b, a) = 0$ if and only if $a = b$;

- $d^*(a, c) \leq d^*(a, b) + d^*(b, c)$.

If $s \in C(\mathbb{N}, A)$ is a sequence and $\{s_n \in C(\mathbb{N}, A) : n \in \mathbb{N}\}$ is a sequence of elements from $C(\mathbb{N}, A)$, then:

- $s = lim_{n\to\infty} s_n$ if $s(i) = lim_{n\to\infty} s_n(i)$ for any $i \in \mathbb{N}$ (pointwise convergence);

- $s = lu - lim_{n\to\infty} s_n$ if $lim_{n\to\infty} d^*(s, s_n) = 0$ (uniform convergence)

- $s = u - lim_{n\to\infty} s_n$ if $lim_{n\to\infty}(d^*(s, s_n) + d^*(s_n, s)) = 0$ (uniform convergence).

If the topology of A is generated by the quasi-metric d, then from $s = u - lim_{n\to\infty} s_n$ it follows that $s = lim_{n\to\infty} s_n$.

Fix $s \in C(\mathbb{N}, A)$. The set

$$Sup(s) = s(\mathbb{N}) \setminus \{\varepsilon\}$$

is the support of the sequence s,

$$l^*(s) = max\{i : a_i \neq \varepsilon\}$$

is the duration of the sequence s and

$$l(s) = |\{i : s(i) \neq \varepsilon\}|$$

is the length of the sequence s.

If the set $\{i : s(i) \neq \varepsilon\}$ is infinite, then we put $l(s) = \infty$. Hence $l(s) \leq l^*(s)$. If $s(\mathbb{N}) = \{\varepsilon\}$, then $Sup(s) = \emptyset$ and $l^*(s) = l(s) = 0$.

Denote by $S^*(A)$ the subspace of all sequences $s \in C(\mathbb{N}, A)$ with finite length. The elements of $S^*(A)$ are called strings. If $l(s) = l^(s) = n < \infty$ then we say that s is a canonical string. Let $S(A)$ be the space of all canonical strings.

We have $d^*(a, b) < \infty$ for all $a, b \in S^*(A)$.

If $a, b \in S^*(A)$ and $l^*(s) = n$, then $c = a \cdot b$ is the string with the following properties:

- $c(i) = a(i)$ for $i \leq n$;

- $c(n+i) = b(i)$ for each $i \in \mathbb{N}$.

Then $S^*(A)$ is a semigroup and $(S(A), \cdot)$ is a monoid with the identity ε, where $\varepsilon(i) = \varepsilon$ for each $i \in \mathbb{N}$. We observe that $S(A)$ is not a subsemigroup of the semigroup $S^*(A)$ and that $S(A) \setminus \{\varepsilon\}$ is a subsemigroup of the semigroups $S^*(A)$ and $S(A)$.

If $s \in S^*(A)$ and $l^*(s) = n$, then we put $\Lambda(s) = s(1)s(2)...s(n) \in L^*(A)$. Then $\Lambda : S^*(A) \longrightarrow L^*(A)$ is an isomorphism of $S^*(A)$ onto $L^*(A)$ and $\Lambda(S(A)) = L(A)$.

If d is an integer quasi-metric and $s \in S^*(A)$, then:

- from $s = lu - lim_{n\to\infty} s_n$ it follows that $s = lim_{n\to\infty} s_n$;

- from $s = u - lim_{n\to\infty} s_n$ it follows that $s = lu - lim_{n\to\infty} s_n$ and there exists $k \in \mathbb{N}$ such that $s_n = s$ for any $n \geq k$.

Let $(A, A_0, \leq, \prec)$ be an A_0-space with the topology T. Then the space $C(\mathbb{N}, A)$ in the topology of pointwise convergence is an A_0-space ([18], Theorem 2). If A is a complete A_0-space (injective space), then the space $C(\mathbb{N}, A)$ in the topology of pointwise convergence is a complete A_0-space (injective space) ([18], Theorem 3).

Let G be a monoid. The quasimetric ρ on G is stable if $\rho(x \cdot u, y \cdot v) \leq \rho(x, y) + \rho(u, v)$ for all $x, y, u, v \in G$. The topology $T(\rho)$ generate by a stable quasi-metric on a monoid G is compatible with the multiplication, i.e. the multiplication is continuous relatively to the topology $T(\rho)$.

Let d be quasi-metric on A. Any element $x \in A$ is identified with the string $a_1a_2...a_n...$, where $a_1 = x$ and $a_i = \varepsilon$ for any $i \geq 2$. Thus $A \subset S(A)$. In [8, 9] was proved that on $S(A)$ there exists a quasi-metric $\hat{d}$ with the properties:

- $\hat{d}(x, y) = d(x, y)$ for all $x, y \in A \subset S(A)$;

- $\hat{d}$ is a stable quasi-metric on $S(A)$;

- if ρ is a stable quasi-metric on $S(A)$ and $\rho(x, y) \leq d(x, y)$ for all $x, y \in A$, then $\rho(x, y) \leq \hat{d}(x, y)$ for all $x, y \in S(A)$;

- if d is an integer quasi-metric, then $\hat{d}$ is an integer quasi-metric to.

On quasi-metric space we consider the ordering induced by the distance function: if ρ is a quasi-metric on X, then $x \leq y$ if and only if $\rho(x, y) = 0$.

Proposition 4.1. Let (A, d) be a quasi-metric space. Then:

(i) $(C(\mathbb{N}, A), d^*)$ is a quasi-metric space and $S^*(A)$ is an open subset of $C(\mathbb{N}, A)$;

(ii) If the space A is connected, then the subspace $S^*(A)$ is connected too.

Proof. Assertion (i) is proved in [8, 9]. Fix $n \in \mathbb{N}$. For any $a = (a_1, a_2, .., a_n) \in A^n$ we put $\psi_n(a) = (x_1, x_2, ...) \in S^*(A)$, where $x_i = a_i$ for $i \leq n$ and $\leq x_i = \varepsilon$ for $i \geq n+1$. Then $\psi_n : A^n \longrightarrow S^*(A)$ is a continuous mapping and $\psi_n(A^n) \subset \psi_{n+1}(A^{n+1})$. Assume that the space A is connected. Since $S^*(A) = \cup\{\psi_n(A^*) : n \in \mathbb{N}\}$ the space $S^*(A)$ is connected too.

Corollary 4.2. Let A be an Alexandroff space. Then $S^*(A)$ and $S(A)$ are Alexandroff spaces too.

Proposition 4.3. Let (A, d) be a quasi-metric space, $r > 0$, $a, b \in A$ and $B(a, r, d) \cap B(b, r, d) = \emptyset$. Then there exists a family $\{U_\mu : \mu \in M\}$ of open non-empty subsets of $C(\mathbb{N}, A), d^*)$ such that:

1. $|M| \geq 2^{\aleph_0}$ and $U_\mu \cap U\eta = \emptyset$ for any distinct elements $\mu, \eta \in M$. For any $\mu \in M$

there exists a point $c_\mu \in C(\mathbb{N}, A)$ such that $U\mu = \{y \in C(\mathbb{N}, A) : d^*(c_\mu, y) < \infty\}$.

2. If (A, d) is a space, then the sets U_μ are connected and the set $U(x) = \{y \in C(\mathbb{N}, A) : d^*(x, y) < \infty\}$ is connected for any $x \in C(\mathbb{N}, A)$.

3. If d is a metric, then $C(\mathbb{N}, A) = \cup\{U_\mu : \mu \in M\}$.

Proof. There exists a family $\{N_\mu : \mu \in M\}$ of infinite subsets of $\mathbb{N}$ such that $|M| \geq 2^{\aleph_0}$ and $N_\mu \cap N\eta$ is a finite set for any distinct elements $\mu, \eta \in M$.

For any $\mu \in M$ we construct the point $c_\mu = (c_{(\mu,1)}, c_{(\mu,2)}, ..., c_{(\mu_n)}, ...) \in C(\mathbb{N}, A)$ such that $c_{(\mu,i)} = a$ for $i \in N_\mu$ and $c_{(\mu,j)} = b$ for $j \in \mathbb{N} \setminus N_\mu$. We put $U_\mu = \{x \in C(\mathbb{N}, A) : d(c_\mu, x) < \infty\}$.

Fix two distinct elements $\mu, \eta \in M$. Assume that $x = (x_1, x_2, ..., x_n, ...) \in U_\mu$. Then the set $P = \{i \in \mathbb{N} : d(c_{(\mu,i)}, x_i) \geq r$ is finite. Hence the set $N_\eta \setminus P$ is infinite, $d(c_{(\mu,i)}, x_i) < r$ for $i \in N_\mu$ and $d(c_{(\eta,i)}, x_j) \geq r$ for $j \in N_\eta \setminus P$. Thus $x \notin U_\eta$. Moreover, $d(c_\mu, c_\eta) = \infty$. Assertion 1 is proved.

Assume that the space (A, d) is connected. Then for any $y = (y_1, y_2, ..., y_n, ...) \in C(\mathbb{N}, A)$ the set $Fin(y) = \{z = (z_1, z_2, ..., z_n, ...) \in C(\mathbb{N}, A) : |\{i\mathbb{N} : y_i \neq z_i\}| < \aleph_0\}$ is connected. Fix $x = (x_1, x_2, ..., x_n, ...) \in C(\mathbb{N}, A)$. The set $U(x)$ is open. suppose that the set $U(x)$ is not connected and U, V are two disjoint non-empty open subsets of $C(\mathbb{N}, A)$ such that $U(x) = U \cup V$. Let $x \in U$. Fix $y \in V$. Then $x \in ClFin(y)$ and, since $y \in Fin(y)$ and $Fin(y)$ is connected, we have $Fin(y) \subset V$. Then $U \cap Fin(y) = \emptyset$, $x \in U$ and $x \notin clFin(y)$, a contradiction. Assertion 2 is proved.

Assume that d is a metric. In this case M is a maximal subset of $C(\mathbb{N}, A)$ such that $d^*(x, y) = \infty$ for any distinct points $x, y \in M$. From the proof of Assertion 1 it follows that $|M| \geq 2^{\aleph_0}$. In this case $U(x) \cap U(y) = \emptyset$ for distinct points $x, y \in M$. Assertion 3 is proved. The proof is complete.

5. Applications

Distinct poset structures have been introduced to accommodate the needs of information theories. In the 1960's, Dana Scott introduced continuous lattices [38, 39, 40, 42, 43, 41] into computer science as a means of providing mathematical models for a system of types that justify recursive definitions of these types. In time, the order theoretic models Scott and others considered evolved into what we now call domains (see [1, 21, 47], RS). The level of abstraction required to understand domain theory remained an obstacle to its widespread use. To remedy this problem, Scott imported from logic the notion of an information system to provide a set-theoretic approach to domains [43]. In this setting, every information system gives rise to a domain in a canonical way. The Hoare powerdomain is an order-theoretic analog of the power set and is used in programming semantics as a model for angelic nondeterminism (see, for example, Plotkin [33]). Some topological aplications in Computer Science are examined in [27, 28, 13, 20, 46, 48].

A poset P is said to be directed-complete if the join ofevery directed subset of P exists in P. A subset S of poset P is a down-set of P provided $S = \{p \in P : p \preceq a \textit{ for some } a \in S\}$. A down-set of P is Scott-closed if it contains the join of each of its directed subsets. An element x of a P is compact if, whenever x is below the supremum of a directed subset set S of P, then $x \in \{p \in P : p \preceq a \text{ for some } a \in S\}$. We use $K(P)$ to denote the subposet of compact elements of P. A directed-complete poset P is algebraic if, for all $p \in P$, the set $K(p) = \{x \in P : x \preceq p\} \cap K(P)$ is directed and $p = \vee K(p)$. We use the term "domain" for an algebraic poset in which the meet of every non-empty subset exists. We will let $\Gamma(P)$ denote the set of all Scott-closed subsets of the directed-complete poset P, ordered by set-inclusion. It is easy to see that $\Gamma(P)$ is closed with respect to finite set-unions and arbitrary set-intersections. Hence $\Gamma(P)$ is the family of closed sets for a topology on P, called the Scott topology on P. The lattice of non-empty Scott-closed subsets of a domain D is called the Hoare powerdomain of D [24].

A domain representation of a topological space X is a function, usually a quotient map, from a subset of a domain onto X (see [5]). The theory of domains was improved by Yu. L. Ershov [17, 19, 18, 16] and now is called the Scott - Ershov theory of domains.

Definition ([24]). An information system is a triple $\mathcal{S} = (S, Con, \vdash)$ consisting of:

(1) a set S whose elements are called propositions or tokens;

(2) a non-empty subset Con of the set of all finite subsets $Fin(S)$ of a set S, called the consistency predicate;

(3) a binary relation $\vdash$ on Con, called the entailment relation.

These entities satisfy the following axioms:

$(IS1)$. Con is a down-set S of $Fin(S)$ with respect to set-inclusion such that $\cup Con = S$.

$(IS2)$. if $A \subset Con$ and $B \subset A$, then $A \vdash B$.

$(IS3)$. if $A, B, C \in Con$, $A \vdash B$, and $B \vdash C$, then $A \vdash C$.

$(IS4)$. if $A, B, C \in Con$, $A \vdash B, andA \vdash C$, then $B \cup C \in Con$ and $A \vdash (B \cup C)$.

Axiom $(IS1)$ implies that every singleton subset of S is a member of Con and that whenever $A \in Con$ and $B \subset A$, then $B \in Con$. Axioms $(IS2)$ and $(IS3)$ imply that $(Con, \vdash)$ is a preordered set, that is, $\vdash$ is a reflexive and transitive relation on Con. The above definition of an information system is differently from the definitions of Scott [43], Davey and Priestly [10], Droste and Göbel [14].

A workload [25, 26] is a triple $W = (D, A, \mathcal{Q})$, where D is the domain, A is a finite subset of the domain (dataset, or instance), and $\mathcal{Q} \subset 2^D$ is the set of queries, that is, some specified subsets of D. Answering a query $Q \in \mathcal{Q}$ means listing all datapoints $a \in A \cap Q$.

A (dis)similarity measure on a set D is a function of two variables $s : D \times D \longrightarrow R$, possibly subject to additional properties. A range similarity query centered at $a \in D$ consists of all $x \in D$ determined by the inequality $s(a, x) < k$ or $s(a, x) > k$, depending on the type of similarity measure. A similarity workload is a workload whose queries are generated by a similarity measure. The formula $d(a, b) = s(a, a) - s(a, b)$, $a, b \in A$ isa distance. In many cases $d(a, b)$ is a quasi-metric. By instance, applied to the similarity measure given by BLOSUM62, as well as of most other matrices from the BLOSUM family, is a quasi-metric on A.

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