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PROFESSOR ALEXANDRU ȘUBĂ ON HIS 70TH BIRTHDAY

Alexandru Șubă is University Professor, Doctor Habilitatus in Mathematical and Physical Sciences. He is a Moldovan mathematician and a remarkable leader of the Moldovan School of Differential Equations, who contributed a lot to the qualitative theory of differential equations and to the education of new generations of highly-qualified specialists. On December 2, 2023, Professor Alexandru Șubă celebrated his 70th anniversary.

He was born in the village of Dănceni from the district of Ialoveni, Republic of Moldova. In 1969 he finished the elementary school from the village of Dănceni; then, in 1971 he finished the secondary school from the town of Ialoveni and in 1976 he graduated from the Faculty of Physics and Mathematics of Moldova State University from Chișinău. At the same time, in 1976 he started his Candidate Degree (equivalent of PhD Degree) at the Institute of Mathematics and Computer Sciences of the Academy of Sciences of Moldova (specialty 01.01.02 – Differential Equations).

In 1982, Alexandru Șubă defended his Candidate Degree thesis in Mathematical and Physical Sciences at the State University of Sankt-Petersburg, Russia. He did it under the supervision of the well-known mathematician Academician Constantin Sibirschi. In 1999 he defended his Doctor Habilitatus Degree thesis (2nd PhD thesis) in Chișinău at

Moldova State University, Institute of Mathematics and Informatics of the Academy of Science of Moldova.

The professional activity of Professor Alexandru Șubă belongs to three institutions: Institute of Mathematics and Informatics of the Academy of Sciences of Moldova (1976–1990, 2010–present) which merged with State University of Moldova in 2022, State University of Moldova (1990 – 2010) and from 1997 to Tiraspol State University, located in Chișinău, which merged with "Ion Creangă" State Pedagogical University in 2022.

Within the Institute of Mathematics and Informatics of the Academy of Sciences of Moldova, the professional activity of Professor Alexandru Șubă evolved as follows: Collaborator of the Laboratory (1976–1981), Scientific Researcher (1981–1985), Senior Scientific Researcher (1985–1990), Deputy Director (2010–2015), head of the Laboratory of Differential Equations (2015–2019), Principal Scientific Researcher (2006–present), while at Moldova State University (Chair of Differential Equations) his career took place as follows: Associate Professor (1990–2000) and University Professor (2001–2010).

Since 1991, Professor Alexandru Șubă has been working fruitfully at Tiraspol State University / "Ion Creangă" State Pedagogical University and lectures to Bachelor, Master and PhD Degree students. In 2006 he won by competition the position of Professor at the Department of Mathematical Analysis and Algebra. The contribution of Professor Alexandru Șubă to the education of new generations of highly-qualified mathematicians is enormous. He supervised scientifically one Doctor Habilitatus thesis in mathematics, 6 doctoral theses in physical and mathematical sciences and about 40 Bachelor and Master Degree theses – all of them defended. In 2015 he was awarded Doctor Honoris Causa of Tiraspol State University.

The scientific activity of Professor Alexandru Șubă is related to dynamic systems: topological theory, integrability and special orbits. Within this research direction he studied the following problems: the development and systematization of the topological theory of dispersed and semi-dynamic systems; the Dulac integrability problem of dynamical systems; GL(2,R)-orbits problem; the problem of distinguishing between a center and the focus; the problem of classifying differential systems with invariant straight lines.

The main results concerning the topological theory of dynamical systems were published in the monograph [1]. There were elaborated the axioms of dynamic systems without uniqueness (dispersed systems) and systematized their topological theory. For planar semi-dynamical systems, the existence of singular points in the presence of non-wandering points is proved. The structure of minimal pseudo-invariant sets, periodic dot sheets and the semi-dynamical systems of characteristic 0 was studied.

Another research direction of Professor Alexandru Șubă concerns the problem of distinguishing between a center and a focus (the problem of the center) for polynomial differential systems

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y) \tag{1}$$

having a singular point $O(0, 0)$ with pure imaginary eigenvalues (of a center or a focus type), where $P(x, y)$ and $Q(x, y)$ are real and coprime polynomials in the variables x and y of degree n, $n = \max\{\deg P, \deg Q\}$. The importance of this problem in the qualitative theory of differential equations arose as part of the still unsolved 16th Hilbert problem and it remains to be one of the most difficult problems to be solved of the list given by Hilbert [2] at the beginning of the past century.

For the first time in the papers of Professor Alexandru Șubă it was proposed a new approach to the problem of the center by simultaneously taking into account the invariant algebraic curves (exponential factors), the focus quantities and Darboux integrability [3], [4]. This result is an improvement of the classical Darboux integrability theorem and leads to the notion of a *center sequence* or, in a more general form, a *center pair*.

We say that a pair of two numbers $(M; N)$ is a *center pair* for (1) if the existence of M invariant algebraic curves (exponential factors) and the vanishing of the focus quantities $L_k,\ k = 1, \dots, N$ imply the singular point $O(0, 0)$ to be a center for (1).

Till present, for polynomial differential systems with irreducible non-homogeneous invariant algebraic curves (exponential factors) the following center sequences are known [3], [4]:

$$\left(\frac{n(n+1)}{2}; 0\right), \left(\frac{n(n+1)}{2} - 2; 1\right), \dots, \left(\frac{n(n+1)}{2} - \left[\frac{n+1}{2}\right]; \left[\frac{n-1}{2}\right]\right)$$

The problem of center sequences was solved completely for some classes of cubic differential systems ($n = 3$) with a given number of invariant straight lines ($l_j \equiv a_j x + b_j y + c_j = 0$), four invariant straight lines ($j = 1, 2, 3, 4$), three invariant straight lines ($j = 1, 2, 3$). In this way, in the period 1992-2005 it is shown that $(l_j, j = 1, 2, 3, 4; N = 2)$ and $(l_j, j = 1, 2, 3; N = 7)$ are center sequences [5], [6].

In the last years, this direction of investigation was highly appreciated by many mathematicians and the obtained results were cited in several papers, see for instance Christopher and Llibre [9], Chavarriga, Giacomini and Giné [7], Chavarriga and Grau [8], Cozma [10], Garcia and Giné [11], Giné [12], Romanovski and Shafer [13].

Concerning the Dulac integrability problem of dynamical systems, the problem of the existence of a center in the sense of Dulac was completely solved by Professor Șubă for plane cubic differential systems having a singular point with one zero eigenvalue [14].

The investigation of the orbits of a differential system belongs to the theory elaborated by Professor Mihail Popa, Doctor Honoris Causa of Tiraspol State University (2013) and refers to the interaction of Lie algebras, systems of differential equations and their algebraic invariants [15]. Professor Alexandru Șubă proved that the $GL(2, R)$–orbit dimension of any polynomial differential system is different from one. He proposed a classification of polynomial differential systems with respect to the dimensions of $GL(2, R)$–orbits [16], [17].

At present, the activity of Professor Alexandru Șubă is focused on the study of polynomial differential systems with multiple invariant straight lines (see, for example, [18], [19]). In this direction, in addition to some important results, he also formulated some problems to be solved in the future. Here we bring only a few of them.

Denote by $M(n)$ (respectively, $M_\infty(n)$) the maximal multiplicity of affine invariant straight lines (respectively, the line at infinity) in the class of polynomial differential systems of degree n. For affine invariant straight lines we have $M(2) = 4$, $M(3) = 7$, $M(4) = 10$ and the evaluation [20]:

$$3n - 2 \leq M(n) \leq 3n - 1, \ n \geq 2$$

Problem 1 (Conjecture 1). $M(n) = 3n - 2$?

Problem 2. *Is it linear the equation of trajectories of each polynomial differential system (1) which has an affine invariant straight line of the maximal multiplicity* $M(n)$ *?*

Problem 3. *Is it true the following equality* $M_\infty(n) = 3n-2$ *? Are linear and has only one affine invariant straight line of multiplicity one the systems for which the line at infinity* $\mathcal{L}_\infty$ *is of maximal multiplicity* ($M_\infty(n) \geq 3n - 2, \ n > 3$) ?

Problem 4. *Is* $M_\infty(n) = 2n + 1$ *the maximal multiplicity of the line at infinity in the class of polynomial differential systems of degree n without affine invariant straight lines ?*

Professor Alexandru Șubă is the author of over 160 scientific publications, published in prestigious journals from the USA, Italy, Spain, Ukraine, Belarus, China, Romania, among them being one monograph, four text books for Bachelor and Master Degree students. He contributed to the organization and development of research in the field of differential equations, founding, by training highly qualified personnel, a scientific school related to the theory of integrability of systems of differential equations.

He worked within two international grants (Canada-France-Moldova, 1999–2001; USA-Moldova, 2001–2003), one European project FP7-PEOPLE-2012-IRSES (2012–2016) and two national projects (2011–2014, 2015–2019). Due to his prestige in the field of mathematics, he became member of the editorial boards of four accredited journals and co-president of the Seminar on Differential Equations and Algebras at Tiraspol State University/"Ion Creangă" State Pedagogical University. The Seminar works on regular basis since 2002, and it is designed for Bachelor, Master and PhD Degree students and scientific researches.

The special appreciation of his scientific work brings him several prizes, titles and medals, namely: prize "Academician Constantin Sibirschi" (2013); Medal "Nicolae Milescu Spătaru" (2019); Medal "Dimitrie Cantemir" (2023); Medal "Ion Creangă" (2023) given by „Ion Creangă" State Pedagogical University. Professor Alexandru Șubă is also Honorary citizen of the village of Dănceni from Ialoveni District (2023).

The present volume is dedicated to Professor Alexandru Șubă at the age of 70, very active in the academic community, full of vigor and optimism, who brought a significant contribution to the development of mathematics in the Republic of Moldova.

On the occasion of his 70th birthday we congratulate Professor Alexandru Șubă on his achievements and we wish him many returns of the day, good health, all the blessings of life, new scientific accomplishments and fruitful didactic activities.

Happy Anniversary, dear Professor Alexandru Șubă!

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