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Dedicated to Professor Alexandru Șubă on the occasion of his 70th birthday

Discovering the mysteries of Pi number using AR technologies

Inga Titchiev, Olesea Caftanatov, and Dan Talambuta

Abstract. The integration of Augmented Reality (AR) in education requires a strategic approach in order to ensure effectiveness in the learning process. AR technologies are constantly evolving, offering new possibilities for educational content, making it an evolving and innovative tool for educators. Exploring the mathematical world of Pi through AR can be an engaging and interactive experience for learners. This article presents the approach used in the development of an Augmented Reality application intended for Pi learning.

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Keywords: Augmented Reality, challenges, education, Pi number.

Descoperirea misterelor numărului Pi utilizând tehnologiile RA

Rezumat. Integrarea Realității Augmentate (RA) în educație necesită o abordare strategică pentru a asigura eficacitatea procesului de învățare. Tehnologiile RA evoluează constant, oferind noi posibilități pentru conținutul educațional, făcându-l un instrument evolutiv și inovator pentru profesori. Explorarea numărului Pi prin RA poate fi o experiență captivantă și interactivă pentru instruiți. În acest articol este prezentată abordarea utilizată în dezvoltarea unei aplicaţii de Realitate Augmentată destinată învățării numărului Pi.

Cuvinte-cheie: Realitate Augmentata, provocări, educație, numărul Pi.

1. Introduction

Augmented Reality (AR) technology in education offers transformative opportunities to enhance learning experiences by overlaying digital content onto the physical world. AR creates immersive and interactive learning environments that engage students by bringing abstract concepts to life, making learning more engaging and memorable. It allows students to visualize complex subjects, explore 3D models, and interact with digital content, making abstract concepts more tangible and easier to understand.

AR applications [4] can be tailored to accommodate diverse learning styles, allowing students to learn at their own pace and providing personalized learning experiences.

By merging the physical and digital worlds, AR captures students' attention, fostering curiosity, and promoting active participation in the learning process.

Moreover, Augmented Reality facilitates interdisciplinary learning by connecting various subjects, enabling students to explore the connections between different fields of study in a more interactive manner. It helps develop critical thinking, problem-solving, and spatial reasoning skills as students engage in interactive AR experiences that require analysis and decision-making. AR bridges the gap between theory and practice by simulating real-world scenarios, allowing students to apply their knowledge in practical contexts.

In this article we will present how AR technology can be used to explore the world of Pi by offering a dynamic and immersive approach to learning mathematics. By integrating AR experiences into educational settings, students can engage with Pi's concepts in creative and interactive ways, fostering a deeper appreciation for its significance in mathematics and beyond.

From ancient civilizations, where approximations of Pi were etched into clay tablets, to the modern era of supercomputers and advanced mathematical theories, the quest to understand Pi has been an enduring journey. The intrigue surrounding Pi has led to profound discoveries about the nature of numbers, the limits of calculation, and the beauty inherent in mathematical patterns. Thus, we decided to develop an augmented mobile application that would be used as a tool in learning this transcendental number.

The Pi Journey App that we developed is an immersive exploration of the mysteries and intricacies of the mathematical constant π (Pi) using cutting-edge AR technology. This unique experience takes users on a visual and interactive journey, unraveling the significance of Pi in a three-dimensional augmented space.

2. Challenges associated with using AR technology in education

However, integrating AR into education faces challenges like infrastructure limitations, content creation complexities, teacher training needs, and ensuring equitable access for all students. Overcoming these hurdles requires investment in resources, professional development, and a commitment to adapting pedagogical approaches to harness the full potential of AR in education.

In order to develop a useful, interactive and efficient application, the following aspects are taken into consideration:

- Defining learning goals and objectives that align with the curriculum or educational outcomes where AR can enhance understanding or engagement.
- Identifying subjects or topics where visualizing, interacting with 3D models, or experiencing immersive content can significantly benefit student.

- Elaborating user-friendly AR application for both educators and students that align with educational goals.
- Developing AR content that supports the learning objectives. This might involve creating 3D models, animations, or utilizing existing AR resources.
- Integrating AR activities into the curriculum to complement and enhance traditional teaching methods, but not as standalone activities.
- Testing the effectiveness of AR in enhancing learning outcomes. In order to refine the approach feedback from both teachers and students are gathered.
- Developing assessment methods to measure the impact of AR on learning outcomes. To continuously improve AR content and teaching strategies feedback can be used.
- Sharing best practices and success stories in integrating AR, encouraging collaboration and innovation.
- Improving and updating AR content and methodologies continuously.

By following these steps and integrating AR strategically, educators can leverage this technology to create immersive, engaging, and effective learning experiences for students across various subjects and educational levels.

3. The mysteries and importance of the mathematical constant Pi

In the realm of mathematics, few constants have captured the imagination and curiosity of scholars and enthusiasts alike as much as the mysterious and revered number π (Pi) [6]. Defined as the ratio of a circle's circumference to its diameter, pi is an irrational and transcendental number with a decimal representation that stretches into infinity without repeating. As a fundamental constant, Pi plays a pivotal role in a myriad of mathematical equations, geometry, and scientific principles, transcending its utilitarian purpose to become a symbol of mathematical beauty and intrigue.

The enigma of Pi lies not only in its seemingly infinite and non-repeating decimal expansion but also in its ubiquitous presence across diverse mathematical landscapes. Its significance extends far beyond the simple geometry of circles, permeating areas such as calculus, trigonometry, and even physics. Pi has become a symbol of mathematical elegance and complexity, challenging mathematicians throughout history [7] to explore its mysteries and pushing the boundaries of mathematical knowledge.

In this exploration, we will delve into the mysteries of Pi, unravelling its infinite decimals, exploring its irrationality and transcendence, and discovering its unexpected appearances in various mathematical and scientific realms. Beyond its numerical significance, we will also delve into the cultural, artistic, and philosophical dimensions of Pi,

examining how this mathematical constant has left an indelible mark on human thought and creativity. For this purpose, we designed 10 cards.

The mysterious nature of Pi lies in several intriguing aspects:

- **Irrationality**: Pi is an irrational number, meaning it cannot be expressed as a simple fraction. Its decimal representation goes on forever without repeating, and it cannot be precisely represented by any finite ratio of integers. This property was proven by Johann Lambert in 1768.
- **Transcendence**: Pi is not only irrational but also transcendental. This means that Pi is not the root of any non-zero polynomial equation with rational coefficients. Ferdinand von Lindemann established the transcendence of Pi in 1882. The combination of irrationality and transcendence makes Pi particularly mysterious in the realm of mathematics.
- **No Discernible Pattern**: Despite extensive computation and exploration, mathematicians have not discovered a discernible pattern or sequence within the digits of Pi. The randomness and lack of repetition in its decimal expansion contribute to the mystery surrounding this mathematical constant.
- **Computational Challenges**: The quest to calculate Pi to as many digits as possible has been ongoing throughout history. From manual calculations to modern supercomputers, mathematicians and computer scientists continually strive to push the boundaries of Pi's decimal expansion. Calculating Pi to a high degree of precision poses computational challenges. While modern computers have calculated Pi to trillions of digits, the process remains resource-intensive, emphasizing the vastness and complexity of Pi's decimal expansion.
- **Cultural and Philosophical Significance**: Pi has cultural and philosophical significance beyond its mathematical properties. Its mysterious and infinite nature has inspired contemplations about the limits of human knowledge and the nature of mathematical reality.

The exploration of Pi's digits continues to be a captivating pursuit in the field of mathematics and not only. Through, Pi Journey application we intend to express a part of people's passion to this mysterious number.

4. Some consideration in Pi application development

AR fosters collaboration and teamwork as students engage in shared AR experiences, encouraging peer-to-peer learning and cooperation. It has the potential to make learning

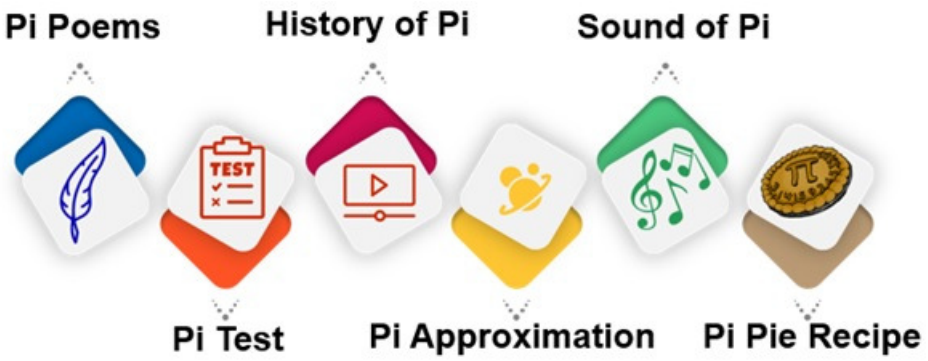

Figure 1. Mysteries of Pi App

more accessible for students with diverse learning needs [5] by providing alternative ways to access and interact with educational content.

In order to facilitate learning, increase study efficiency and successfully adapt to the multitude of learning situations, it is necessary to determine the specific preferences of the personal learning style so that they can be applied in a targeted manner.

Stable individual differences in the way of learning affect the rhythms and quality of learning, and especially determine the option for one or another learning strategy as one's own and personal way of approaching a learning situation.

In the application development the VAK (Visual, Auditory, Kinesthetic) learning style theory was applied.

For each category of learners the following discovery Pi activities were proposed:

- **Visual Learners**: Graphical Representations like diagrams, charts, and visual aids to illustrate Pi's relationship with circles, showing geometric models and patterns visually. Video displaying Pi's digits, sequences, or relationships with shapes, allowing exploration through interactive visual elements.
- **Auditory Learners**: Explanations or discussions about Pi's significance, history, and applications in an auditory format, such as podcasts or recorded lectures. Pi Chants or Songs, in particular Pi Symphony by Lars Ericksone.
- **Kinesthetic Learners**: Engage learners in measuring circles, calculating circumferences, and experimenting with circular objects to explore Pi's mathematical properties practically.

4.1. Designing augmented Pi artifacts and Pi markers

An augmented artifact and marker has quite a few tasks to accomplish [1]. Besides the fact that it has to capture student's attention, entice them to pick up their mobile and scan

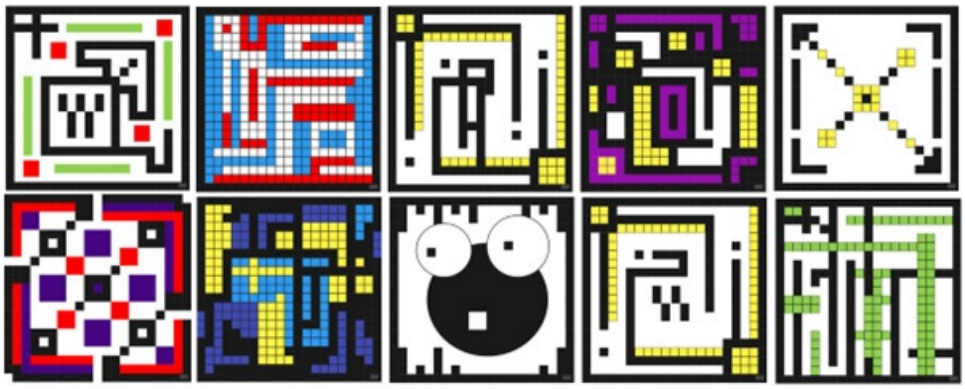

Figure 2. Augmented Pi artifacts and Pi markers

the image - it should have a high quality to let the AR experience come to life. Therefore, in this section, we will describe the best practice of designing augmented artifacts and markers that we observed as a result of the working process and testing.

Moreover, we will describe our experiences with low and high star rating image targets. We consider that "markers" are the digital form of image targets that Vuforia Engine can detect and track by comparing extracted natural features from the camera image against a known image target resources database.

Markers come in various forms: simple, flat image targets, curled targets in the form of cylindrical shapes, or multi-targets in the composition of a box. We define "artifacts" as the physical form of markers. They can also come in various forms: cards, papers, newspapers, posters, objects, etc. In our cases, it is a matte laminated image with size 10 x 10 cm, see Figure 1. The main purpose of the artifact is to trigger the augmentation content when it is scanned by camera.

In the next section we will give some examples of these Augmented Pi Artifacts.

4.2. Pi Artifact that explains the relationship between Pi and the circumference of a circle

The relationship between Pi (π) and the circumference of a circle is defined by a fundamental geometric formula. The circumference (C) of a circle is calculated using Pi and the circle's diameter (D) or radius (r). The formula for the circumference of a circle is as follows:

$$C = 2\pi r \tag{1}$$

or equivalently

$$C = \pi D, \tag{2}$$

where C is the circumference of the circle, π (Pi) is the mathematical constant approximately equal to 3.14159, r is the radius of the circle, D is the diameter of the circle.

This relationship is derived from the definition of Pi, which represents the ratio of a circle's circumference to its diameter. Essentially, Pi is the constant that relates the size of a circle to its "wrap-around" distance. The formula (1) expresses that the circumference is equal to twice the product of Pi and the radius. Alternatively, the formula (2) emphasizes that the circumference is equal to Pi multiplied by the diameter. Since the diameter is twice the radius, these two formulas are equivalent.

The augmented reality Pi artifacts utilize graphical representations to elucidate the correlation between Pi and the circumference of a circle, catering to diverse learning styles through visual and interactive elements.

4.3. Pi Artifacts that represents digits as melodic elements

Pi's influence extends into the realm of music, where musicians and composers have explored creative ways to incorporate the mathematical constant into their works. Pi-themed compositions showcase a unique fusion of mathematics and art, offering a creative interpretation of this transcendental number.

Musicians have experimented with using the digits of Pi as melodic elements in their compositions. Assigning musical notes or intervals to the numerical digits allows for the creation of melodies that directly reflect the numerical sequence of Pi. Composers have employed Pi to structure the rhythmic and harmonic elements of their music. For example, using the digits of Pi to determine the length of musical phrases, the arrangement of sections, or the timing of specific musical events can result in compositions with a distinct mathematical foundation.

We designed an artifact that contains a playlist with the following musics [2]:

- **Pi Symphony by Lars Ericksone**: Lars Erickson, captivated by the enigmatic nature of Pi (π), composed "Pi Symphony" in the early 1990s. Crafting a melody from the seemingly random digits of Pi, Erickson's magnum opus demonstrated that, contrary to expectations, a composition based on Pi could be as majestic as a symphony.
- **Pi Symphony by Jim Zamerski**: Jim Zamerski crafted a melody using 226 digits of π, by utilizing the 12 tones in music as the foundation. While sharing the numerical essence of π with Lars Erickson's piece, Zamerski's composition takes on a lighter ambiance. It initiates with a touch of melodrama, swiftly transitioning into a danceable tune with a dynamic tempo that fluctuates throughout its entirety.

- **Pi Symphony by David Macdonald**: Another example is David Macdonald's "Pi Symphony", which transforms the first 100 digits of Pi into a musical composition. The sequence of digits dictates the pitch, rhythm, and dynamics of the piece, offering an auditory experience that mirrors the mathematical constant. David Macdonald added a diverse angle to Pi-inspired music. Incorporating harmonic elements played by the left hand, Macdonald's composition concealed the randomness of Pi. The poignant piece seemed to transport listeners to a mythical realm in their minds.

This artifact employs auditory elements to cater to a learning style based on sound and hearing.

4.4. Pi poems Artifacts

While Pi (π) is primarily a mathematical constant, its intriguing nature has found its way into literature, where writers and authors have incorporated it as a symbol, metaphor, or even as a theme. Pi poems may use the actual numerical sequence of Pi (3.14159...) to determine elements of the poem, such as the number of syllables in each line or the length of stanzas. For example, the number of syllables in each line might correspond to the digits of Pi (3 syllables for the first line, 1 for the second, 4 for the third, and so on).We dedicated a few artifacts for learning Pi poems in the Romanian Language, i.e. "Iarna lui Pi" by Iuliana Ciubuc, see Table [3]. Pi poem artifacts use the characteristic of visual learning style.

Table 1. Romanian Pi poem with the sequence of Pi 3,141592653589793

Iar	e	rece	E	iarna	adevarata	Cu	zapada	multa	nor	Ceata	cumplita	tematoare	Troiene	visoclite	are
3	1	4	1	5	9	2	6	5	3	5	8	9	7	9	3

4.5. Pi Artifacts for visualizing and learning the first 100 decimal digits

Creating Pi artifacts for visualizing and learning the first 100 decimal expansions of Pi can be an engaging way to explore this mathematical constant. In Figure 2, two instances are depicted: in the left image, the user correctly inputs the first 8 decimal digits; while in the right image, the user inaccurately inputs the 9th digit, resulting in a red highlighted input section.

4.6. Pi Artifacts for assessing knowledge of the number Pi

For the evaluation of knowledge about the number Pi, a scenario was developed that represents a five-item test. After answering to them, feedback is given about which items were answered correctly and which were incorrect.

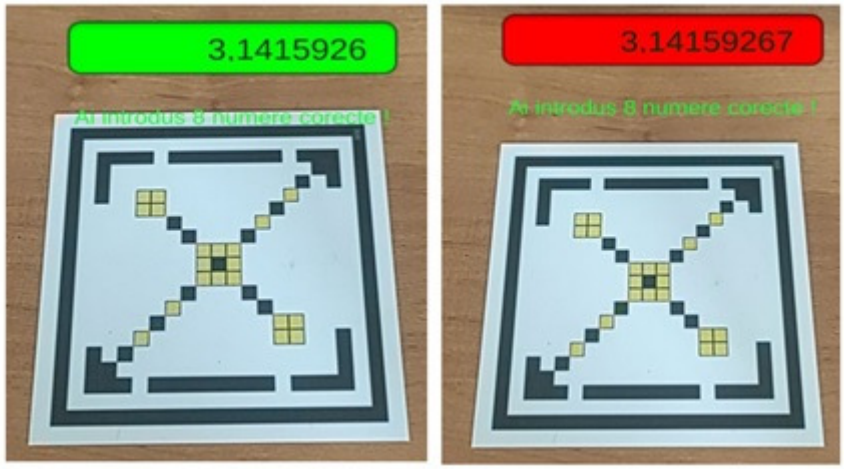

Figure 3. Pi artifacts for visualizing and learning the first 100 decimal digits

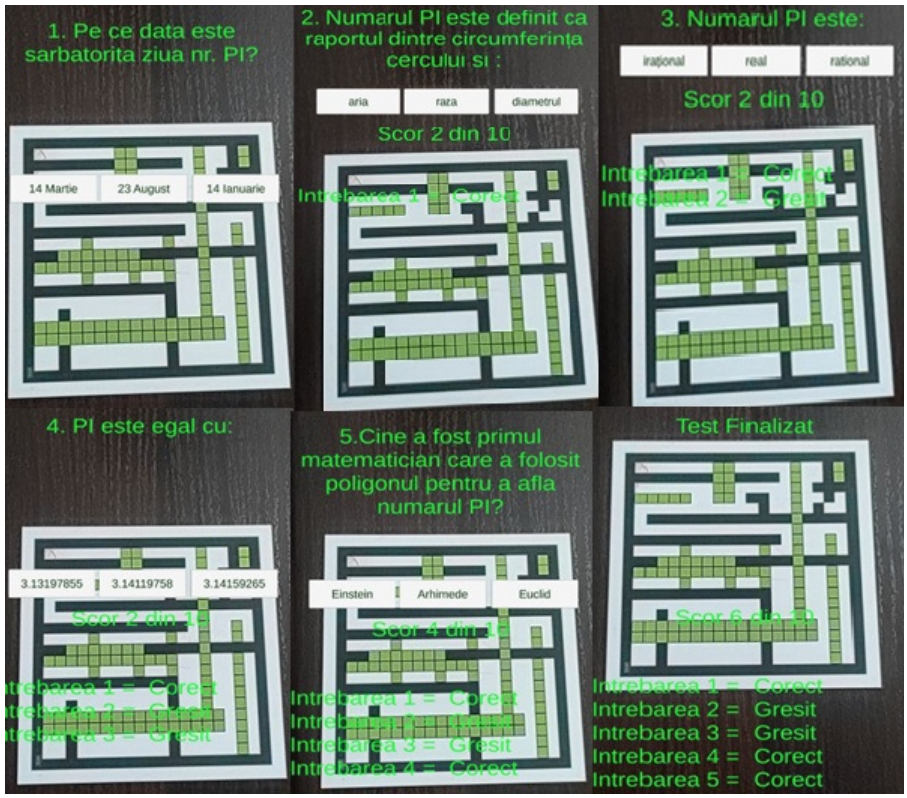

Figure 4. Test for assessing knowledge of the number PI

5. Conclusions

In this article we will present how AR technology can be used to explore the world of Pi by offering a dynamic and immersive approach to learning mathematics. By integrating AR experiences into educational settings, students can engage with Pi's concepts in creative and interactive ways, fostering a deeper appreciation for its significance in mathematics and beyond.

The educational Application Pi Journey is delivered via mobile device that engages pupils with a wide range of multi-sensory learning experiences, provide rich, contextualized learning for understanding the concepts related to transcendental number Pi.

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